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Measuring unfair (in)equality ☆

Ingvild Almås [a,b,*], Alexander W. Cappelen [a], Jo Thori Lind [b], Erik Ø. Sørensen [a], Bertil Tungodden [a,c]

[a] Norwegian School of Economics and Business Administration, Bergen, Norway
[b] University of Oslo, Norway
[c] Chr. Michelsen Institute, Bergen, Norway

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ABSTRACT

This paper shows one way of generalizing the standard framework of inequality measurement to allow for a distinction between fair and unfair inequalities. We introduce the unfairness Lorenz curve and the unfairness Gini, which are generalizations of the standard versions of the Lorenz curve and the Gini. With this more general framework in place, we study the implications of responsibility-sensitive theories of justice for the evaluation of the income distribution in Norway from 1986 to 2005. We find that both the pre-tax and the post-tax income distributions have become less fair in Norway, even though the standard Gini for the pre-tax income distribution has decreased in the same period. Two trends explain this development: the increase in income share of the top percentile and the change in the situation of females in the labor market. The concentration of income at the top of the distribution contributes both to increased unfairness and increased inequality, whereas the increase in females' working hours and level of education primarily contributes to a reduction in inequality. Thus, the latter effect dominates for the standard Gini and the former effect for the unfairness Gini. Furthermore, we find that the increase in post-tax unfairness is even larger than the increase in pre-tax unfairness, which shows that the tax system in Norway contributes less to eliminating unfairness in 2005 than in 1986.

1. Introduction

Most people view a strict egalitarian income distribution as unfair. It is evident from the political debate, surveys (Gaertner and Schwettmann, 2007; Schokkaert and Devooght, 2003), economic experiments (Cappelen et al., 2007; Frohlich et al., 2004; Konow, 2000) and contemporary theories of justice (Arneson, 1989; Bossert, 1995; Fleurbaey, 1995; Roemer, 1996, 1998) that people view some inequalities, e.g. inequalities arising from differences in the number of hours worked, as fair, and other inequalities, e.g. inequalities arising from gender or race, as unfair. Hence, we can have both unfair equalities and unfair inequalities, and the question is how we should measure overall unfair (in)equality, or unfairness, in society.

The standard approach to inequality measurement does not make a distinction between fair and unfair inequalities. All inequalities are considered unfair, and any movement towards a more equal distribution is considered an improvement in terms of fairness. Such a movement may take place through eliminating what many consider to be fair inequalities, however, and may thus actually represent a step towards a more unfair society. Therefore, we propose a framework for inequality measurement that allows for alternative viewpoints of what is a fair income distribution. The defining feature of our approach is that, for a given interpretation of a fair income distribution, we measure how much each individual's actual income deviates from what would be his fair income. This can be done both for pre-tax and post-tax incomes, which also allow us to study the extent to which the tax system contributes to a reduction in unfairness.

Alternative approaches to cope with the distinction between fair and unfair inequalities are given in Bourguignon et al. (2007), Devooght (2008), and Roemer et al. (2003). Our work is closest to Devooght (2008), who uses the same general normative framework and employs a similar empirical approach in the study of Belgian income data. But the present study differs from Devooght (2008) in important respects. First, we generalize the standard framework of inequality measurement by introducing the unfairness Lorenz curve and the unfairness Gini as generalizations of their standard counterparts. This makes it straightforward to compare our measures of unfairness and the standard versions of the Lorenz curve and the Gini. In fact, the standard approach is a special case within this framework, in which the fair income of each individual is equal to the average income in society. Second, we study the development of unfairness over time and for an entire population, and we do so both for the pre-tax and post-tax distributions of incomes.

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* Corresponding author. Norwegian School of Economics and Business Administration, Bergen, Norway.
E-mail addresses: ingvild.almas@nhh.no (I. Almås), alexander.cappelen@nhh.no (A.W. Cappelen), j.t.lind@econ.uio.no (J.T. Lind), sameos@gmail.com (E.Ø. Sørensen), bertil.tungodden@nhh.no (B. Tungodden).

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We analyze the implications of responsibility-sensitive theories of justice for the evaluation of the pre-tax and post-tax income distributions in Norway from 1986 to 2005, where a responsibility-sensitive theory of justice specifies what acceptable and unacceptable sources of inequality in society are. We find that both the pre-tax and post-tax income distributions in Norway have become less fair, even though the standard Gini for the pre-tax income distribution has decreased in the same period. We show that this holds for different views on what individuals should be held responsible for, and for alternative specifications of responsibility-sensitive theories of justice.

There are two main explanations of this development: the increase in the top percentile income share and the change in the situation of females in the labor market. The concentration of income at the top of the distribution contributes both to increased unfairness and inequality, whereas the increase in females' working hours and level of education primarily contributes to a reduction in inequality and only marginally reduces unfairness. Furthermore, we find that the increase in post-tax unfairness is even larger than the increase in pre-tax unfairness, which shows that the tax system in Norway contributes less to eliminating unfairness in 2005 than in 1986.

Section 2 generalizes the standard approach to inequality measurement. Section 3 introduces the generalized proportionality principle and the responsibility cut. Section 4 describes the data for Norway and the estimation of the labor earnings equation. The main empirical analysis is presented in Section 5, whereas Section 6 and Section 7 present robustness tests. Section 8 provides some concluding comments.

2. Generalizing the standard framework

In this section, we show how the standard framework of inequality measurement, which measures deviation from a norm income distribution of strict equality, can be generalized to cover other ideas of a fair income distribution. Formally, in this section we assume that any alternative, $\boldsymbol{A}$, contains a set of individuals, $\boldsymbol{N}=\{1,\ldots,n\}$. Each individual, i, is characterized by the pair, $(y_i^{\boldsymbol{A}},z_i^{\boldsymbol{A}})$, where $y_i^{\boldsymbol{A}}\geq 0$ is the actual income and $z_i^{\boldsymbol{A}}\geq 0$ is the fair income of individual i in $\boldsymbol{A}$. Hence, an alternative $\boldsymbol{A}$ is characterized by $\boldsymbol{A}=[(y_1^{\boldsymbol{A}},z_1^{\boldsymbol{A}}),\ldots,(y_n^{\boldsymbol{A}},z_n^{\boldsymbol{A}})]$, where the average income is denoted as $\mu(\boldsymbol{A})=n^{-1}\sum_i y_i^{\boldsymbol{A}}$. In the standard framework, $z_i^{\boldsymbol{A}}=\mu(\boldsymbol{A})$ for all individuals, whereas the more general framework allows for individual-specific fair incomes. We assume, however, that for any $\boldsymbol{A}$, $\sum_i y_i^{\boldsymbol{A}}=\sum_i z_i^{\boldsymbol{A}}$, i.e., the distribution of fair incomes reflects a perception of how total income in society should be distributed. Thus, the set of possible alternatives to consider is given by

$$\Xi = \left\{\boldsymbol{A}\,|\,z_i^{\boldsymbol{A}}\geq 0 \text{ for all } i \text{ and } \sum_i y_i^{\boldsymbol{A}} = \sum_i z_i^{\boldsymbol{A}}\right\}.$$

The classical representation of the standard Lorenz curve, $\mathbf{L}(s;\boldsymbol{A})$, is based on a ranking of individuals according to their actual income, $y_{1(\boldsymbol{A})}\leq y_{2(\boldsymbol{A})}\leq\ldots\leq y_{n(\boldsymbol{A})}$, where $y_{1(\boldsymbol{A})}$ is the person with the lowest actual income in $\boldsymbol{A}$, and so on. But the standard Lorenz curve can equally well be constructed by ranking the individuals according to the difference between their actual income and the average income,

$$\mathbf{L}(s;\boldsymbol{A}) = \frac{\sum_{i=1}^{\lfloor ns\rfloor} y_{i(\boldsymbol{A})}}{n\mu(\boldsymbol{A})} = \frac{\sum_{i=1}^{\lfloor ns\rfloor}\left(y_{i(\boldsymbol{A})}-\mu(\boldsymbol{A})\right)}{n\mu(\boldsymbol{A})} + s,\ 0\leq s\leq 1, \tag{1}$$

where $\lfloor ns\rfloor$ is the largest integer not greater than ns. The second part of Eq. (1) is only a normalization and does not affect the comparison of any two alternatives, and thus it follows that the standard Lorenz curve can be defined on cumulative shares of the difference between the fair income, interpreted as the average income, and the actual income. The two alternative representations of the standard Lorenz curve are shown graphically in Fig. 1.

The difference based approach can easily be generalized to other fairness principles than strict equality, where the unfairness Lorenz curve formally is defined as follows:

$$\mathbf{L^U}(s;\boldsymbol{A}) = \frac{\sum_{i=1}^{\lfloor ns\rfloor} u_{i(\boldsymbol{A})}}{n\mu(\boldsymbol{A})},\quad 0\leq s\leq 1,\ z_i^{\boldsymbol{A}}\geq 0 \text{ for all } i. \tag{2}$$

The unfairness Lorenz curve allows for a more general definition of fair income than the standard Lorenz curve, but keeps the ranking according to unfairness, $u_{1(\boldsymbol{A})}\leq u_{2(\boldsymbol{A})}\leq\ldots\leq u_{n(\boldsymbol{A})}$. In the more general case, this may not be equivalent to ordering individuals by actual incomes. We also observe that the difference based approach provides an attractive normalization of the unfairness Lorenz curve. It implies that $\mathbf{L}(1;\boldsymbol{A}) = \sum_i u_i^{\boldsymbol{A}} = 0$, which nicely captures that the total population as a group cannot be unfairly treated.

The focus on unfairness Lorenz curves can be justified in exactly the same manner as in the standard case. Let us impose the following modified versions of the standard conditions on an inequality partial ordering defined on the alternatives in Ξ, where $\boldsymbol{A}\preceq\boldsymbol{B}$ represents that there is at least as much unfairness in $\boldsymbol{B}$ as in $\boldsymbol{A}$.

Definition 1. Scale Invariance: For any $a>0$ and $\boldsymbol{A},\boldsymbol{B}\in\Xi$, if $\boldsymbol{A} = a\boldsymbol{B}$, then $\boldsymbol{A}\sim\boldsymbol{B}$.

Definition 2. Anonymity: For any permutation function $\rho:\boldsymbol{N}\rightarrow\boldsymbol{N}$ and for $\boldsymbol{A},\boldsymbol{B}\in\Xi$, if $\left(y_i^{\boldsymbol{A}},z_i^{\boldsymbol{A}}\right) = \left(y_{\rho(i)}^{\boldsymbol{B}},z_{\rho(i)}^{\boldsymbol{B}}\right)$ for all $i\in\boldsymbol{N}$, then $\boldsymbol{A}\sim\boldsymbol{B}$.

Definition 3. Generalized Pigou–Dalton: For any $\boldsymbol{A},\boldsymbol{B}\in\Xi$, where $z_i^{\boldsymbol{A}} = z_i^{\boldsymbol{B}}$ for all i, if there exist j,k such that $u_j^{\boldsymbol{A}}<u_j^{\boldsymbol{B}}\leq u_k^{\boldsymbol{B}}<u_k^{\boldsymbol{A}}$ and $u_i^{\boldsymbol{A}} = u_i^{\boldsymbol{B}}$ for all $i\neq j,k$, and $y_j^{\boldsymbol{B}}-y_j^{\boldsymbol{A}} = y_k^{\boldsymbol{A}}-y_k^{\boldsymbol{B}}$, then $\boldsymbol{A}\succ\boldsymbol{B}$.

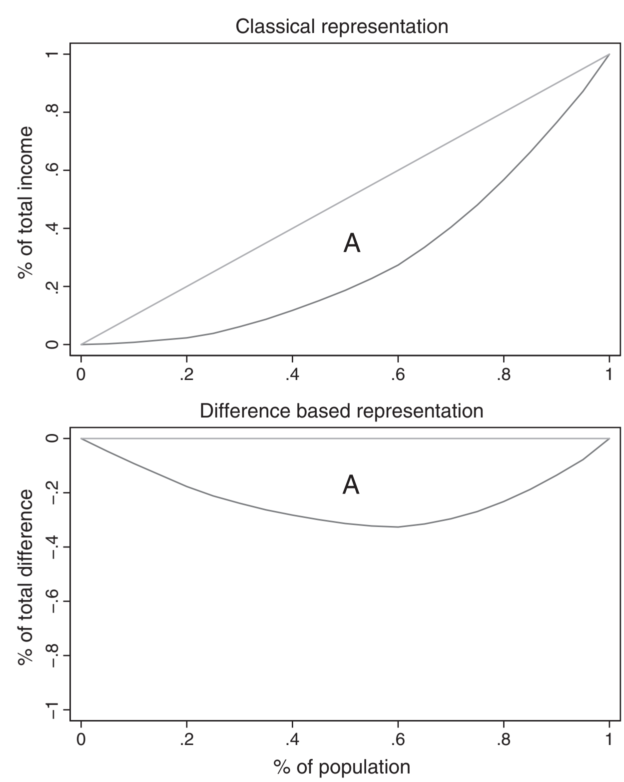

Fig. 1. Two representations of the standard Lorenz curve. The figure displays two representations of the standard Lorenz curve, where the classical representation relies on cumulative income shares and the difference based representation relies on cumulative shares of the difference between the average income and the actual income. The area A is the same in both panels.

Definition 4. Unfairism: For any $\boldsymbol{A},\boldsymbol{B}\in\Xi$ such that $\mu(\boldsymbol{A})=\mu(\boldsymbol{B})$, if $u_i^{\boldsymbol{A}}=u_i^{\boldsymbol{B}}$ for all $i\in\boldsymbol{N}$, then $\boldsymbol{A}\sim\boldsymbol{B}$.

Scale invariance states that if all actual incomes and fair incomes are rescaled with the same factor, then the level of unfairness remains the same. Anonymity states that the ranking of alternatives should be unaffected by a permutation of the identity of individuals. The generalized version of the Pigou–Dalton criterion states that any fixed transfer of income from a person who is less unfairly treated to a person who is more unfairly treated reduces the level of unfairness. Finally, unfairism states that we are only concerned with how unfairly each person is treated, defined as the absolute deviation from the fair income of the individual.

In line with the standard approach, we define the unfairness Lorenz Dominance ($\mathbf{LD}^u$) as follows:

Definition 5. Unfairness Lorenz dominance: For any $\boldsymbol{A},\boldsymbol{B}\in\Xi$, $\boldsymbol{A}\ \mathbf{LD}^u\ \boldsymbol{B}$ if and only if $\sum_{i=1}^{\lfloor ns\rfloor} u_{i(\boldsymbol{A})}^{\boldsymbol{A}}/n\mu(\boldsymbol{A})\geq\sum_{i=1}^{\lfloor ns\rfloor} u_{i(\boldsymbol{B})}^{\boldsymbol{B}}/n\mu(\boldsymbol{B})$ for all $0\leq s\leq 1$, and there exists s such that $\sum_{i=1}^{\lfloor ns\rfloor} u_{i(\boldsymbol{A})}^{\boldsymbol{A}}/n\mu(\boldsymbol{A})>\sum_{i=1}^{\lfloor ns\rfloor} u_{i(\boldsymbol{B})}^{\boldsymbol{B}}/n\mu(\boldsymbol{B})$.

We now note that, as an analogue to the standard framework, unfairness Lorenz dominance is equivalent to a strict ranking of the alternatives for any partial ordering that satisfies the basic conditions.

Observation 1. For any partial inequality ordering on Ξ satisfying Scale Invariance, Anonymity, Generalized Pigou–Dalton and Unfairism: If $\boldsymbol{A}\ \mathbf{LD}^u\ \boldsymbol{B}$, then $\boldsymbol{A}\prec\boldsymbol{B}$.

The proof of the observation is provided in Appendix A.

In this framework, it is also straightforward to modify the standard inequality measures. In our empirical analysis, we focus on the most common of these, namely the Gini, where the unfairness version is defined as

$$\mathbf{G}^u(\boldsymbol{A})=\frac{1}{2n(n-1)\mu(\boldsymbol{A})}\sum_i\sum_j\left|u_i^{\boldsymbol{A}}-u_j^{\boldsymbol{A}}\right|. \tag{3}$$

We study cases where $y_i\geq 0$ for all individuals, and thus it follows that the unfairness Gini has a maximum value of two.[1] It reaches its maximum when the actual income for all individuals except one is zero, and the fair income of the person who has all the income is zero and the fair income for one of the individuals with zero income is the total income in the economy. As illustrated in Fig. 1, the unfairness Gini is represented by the area between the unfairness Lorenz curve and the horizontal axis. In the case where everyone's fair income is equal to the average income, the unfairness Gini equals the standard Gini.

3. What is the fair income distribution?

In order to measure the extent of unfairness in a given situation, we need to specify a principle of fairness that determines what would have been the fair distribution of income in this situation. We are concerned with fairness principles that are responsibility-sensitive in the sense that they justify inequalities due to responsibility factors, but do not justify inequalities due to non-responsibility factors (Arneson, 1989; Cohen, 1989; Roemer, 1996, 1998).

For any given *responsibility cut*, i.e., a partition of the set of pre-tax income determinants into factors for which an individual is and is not responsible, the pre-tax income of an individual, i, can be written as $f(\boldsymbol{x}_i^R,\boldsymbol{x}_i^{NR})$, where $\boldsymbol{x}_i^R$ and $\boldsymbol{x}_i^{NR}$ represent the vector of responsibility and non-responsibility factors, respectively.[2] A responsibility-sensitive fairness principle needs to address two questions. First, how should an individual's fair income, z_i, depend on the vector of responsibility factors, $\boldsymbol{x}_i^R$? Second, where should the cut between $\boldsymbol{x}_i^R$ and $\boldsymbol{x}_i^{NR}$ be drawn? The two questions are discussed separately below.

3.1. A responsibility-sensitive fairness principle

The classical proportionality principle is a well-known responsibility-sensitive fairness principle, where income is distributed in proportion to each individual's claim and where the claim is given by the value of the factor for which the individual is responsible. For example, if we assume that the number of working hours is the only responsibility factor, then a person's claim is equal to the number of hours worked. In this case, the classical proportionality principle assigns to each individual a share of the total income that is equal to his share of the total number of hours worked.

In this paper we apply a generalized version of the classical proportionality principle, as developed in Cappelen and Tungodden (2010). The generalized proportionality principle holds that an individual's claim is given by what would have been the average income in a hypothetical situation where everyone had the same responsibility vector as this individual. Hence, an individual's claim depends on the non-responsibility factors of all the individuals in the economy, but only on the individual's own responsibility factors.

The claim of individual i, $g(\boldsymbol{x}_i^R;\cdot)$, can be written as

$$g\left(\boldsymbol{x}_i^R;\cdot\right)=\frac{1}{n}\sum_j f\left(\boldsymbol{x}_i^R,\boldsymbol{x}_j^{NR}\right). \tag{4}$$

Individual i's fair income is then given by,

$$z_i^{GPP}=\frac{g\left(\boldsymbol{x}_i^R;\cdot\right)}{\sum_j g\left(\boldsymbol{x}_j^R;\cdot\right)}\sum_i y_i. \tag{5}$$

The generalized proportionality principle treats all individuals as if they were identical with respect to all non-responsibility factors. The principle can be said to be egalitarian because it eliminates all inequalities arising from non-responsibility factors, i.e., *unfair inequalities*. The generalized proportionality principle can also be said to be responsibility-sensitive because it preserves inequalities that are only arising from responsibility factors, i.e., *fair inequalities*.

The generalized proportionality principle satisfies the classical minimal requirements of unfair inequality elimination and fair inequality preservation proposed by Bossert and Fleurbaey (1996). First, any two individuals with the same responsibility factors are assigned the same fair income. Second, in any situation where all individuals have the same non-responsibility factors, each individual's fair income is equal to his pre-tax income.[3]

In the simple case where each individual's average productivity is constant and there is only one responsibility factor, the generalized proportionality principle is identical to the classical proportionality principle. In other economic environments, the classical proportionality principle is questionable. First, it sometimes justifies redistribution even if all individuals have the same non-responsibility factor, that is, the classical proportionality principle violates the minimal requirement of fair inequality preservation. Second, it is not well-defined in situations with more than one responsibility factor.

In sum, we consider the generalized proportionality principle an attractive formalization of a responsibility-sensitive fairness principle. There are other responsibility-sensitive fairness principles that satisfy both the minimal requirement of unfair inequality elimination (or some version of it) and the minimal requirement of fair inequality preservation (Fleurbaey, 2008), and we consider three such alternatives in Section 7: two versions of the *egalitarian equivalent* principle and the *conditional egalitarian* principle.

[1] For computational purposes, a simpler formulation of the unfairness Gini is given by $G^u(\boldsymbol{A})=\frac{2}{n(n-1)\mu(\boldsymbol{A})}\sum_i iu_{i(\boldsymbol{A})}$.

[2] Here, and in the following, we suppress the notation $\boldsymbol{A}$ for an alternative.

[3] A complete characterization of the generalized proportionality principle can be made based on the strong requirement of fair inequality preservation and a rather weak requirement of unfair inequality elimination (Cappelen and Tungodden, 2010).

3.2. Drawing the responsibility cut

In the philosophical literature, a prominent response to the question of where to draw the responsibility cut has been that individuals should be held responsible for factors under their control, but not for factors beyond their control (Cohen, 1989). However, other views, such as meritocratism, can be seen as holding individuals responsible for all personal factors, even genetic factors, independently of whether these factors are under individual control or not, and there is some experimental evidence indicating that this position is more prevalent in the population (Cappelen et al., 2007).

We do not focus on how a particular responsibility cut should be justified. Instead we analyze the implications of various responsibility sets within our framework, and leave it to the reader to determine which of these is more appealing. Nonetheless, to simplify the presentation, we begin by focusing on the responsibility cut where a person is held responsible for the number of hours worked, years of education, whether he or she works in the public or private sector, and his or her county of residence, whereas the person is not held responsible for the field of education, age and gender. We do not make the claim that this is the correct responsibility set, and later we analyze the implications of expanding and reducing the responsibility set and of controlling for family background and innate ability for a subsample of the population.

Clearly, the implications of the generalized proportionality principle strongly depend on which factors are included in the responsibility set. To illustrate, if individuals are not held responsible for any factors, then the principle implies that the fair income distribution is to give everyone an equal share of total income. On the other hand, if individuals are held responsible for all factors, then it implies that the fair income distribution is given by the pre-tax income distribution.

In an empirical analysis we need to address the specific question of how to treat the unobservable factors that affect a person's pre-tax income. In the main part of this paper we respond to this question by appealing to the basic egalitarian intuition that deviations from an equal distribution can only be justified if individuals differ with respect to some responsibility factors. In principle, some of the unobservable factors may be responsibility factors, but it is not possible to determine whether individuals differ with respect to these factors. We therefore find it attractive to treat all unobservable factors as non-responsibility factors. However, for the sake of completeness, we also consider the implications of including the unobservable factors in the responsibility set.

4. Descriptive statistics

We apply our framework to study development of the pre-tax and post-tax income distributions in Norway in the period from 1986 to 2005. In this section we describe the data and the estimation of the labor earnings equation.

4.1. Data

We use a database constructed at the Norwegian School of Economics and Business Administration based on administrative records collected by Statistics Norway (Møen et al., 2003). In these records, information on individuals is collected using a unique personal identifier that makes it possible to link data collected for various administrative purposes.

Our measure of pre-tax income is annual labor earnings.[4] It includes all earnings from work activities, but excludes pensions, transfers that are not direct replacements of labor income, and any capital income.[5] However, it includes temporary sick-leave benefits and unemployment benefits. A tax reform in 1992 made income from self-employment part of this definition of labor earnings. This change affected the upper tail of the labor earnings distribution (and increased the variance of labor earnings), but given the relatively small number of self-employed individuals, it barely had any impact on the measured inequality. We deflate all the labor earnings to 1998 prices using the Consumer Price Index constructed by Statistics Norway.

Table 1
Descriptive statistics.

	1986		2005	
Proportion female	0.462		0.485	
Proportion in public sector	0.363		0.392	
Proportion aged 30–39	0.419		0.350	
Proportion aged 40–49	0.342		0.342	
Proportion aged 50–59	0.239		0.307	
Proportion working less than 20 h/week	0.123		0.104	
Proportion working 20–29 h/week	0.136		0.097	
Proportion working more than 30 h/week	0.741		0.800	
	1986		2005	
	Mean	SD	Mean	SD
Years of education	10.87	2.83	12.46	2.68
Labor earnings in thousand of 1998 NOK	219	118	321	243
Number of observations	994,296		1,335,078	

Note: Conversion to 1998 prices is made using the Consumer Price Index.

The number of hours worked in a normal week is reported by employers to the National Insurance authorities as indicator variables for “less than 20 h a week”, “20 to 30 h a week”, and “more than 30 h a week”, and we use this to measure the number of hours worked. Schooling is taken from a national database of education completed, and includes data from the 1970 census and onwards (Vassenden, 1995).

We restrict the analysis to persons with complete data on demographics and education. We also restrict ourselves to those for whom we observe non-zero annual labor earnings and with reported hours of work at the time of observation, end of May 1986–1994 and end of November 1995–2005. In doing so, we not only remove those out of the labor force due to unemployment, disability, or some other reason, but also the self-employed and those working in companies not complying with the reporting guidelines. By far the most important restriction is that we restrict ourselves to persons with reported hours of work. In 1986, 27% of all men and 35% of women had no reported hours of work, while these same figures in 2005 were 26% and 29%. The other restrictions only remove a couple of additional percentage points, and this is constant over the period. Summing up, the restrictions make men slightly more represented in the analysis (as shown in Table 1).

For parts of the analysis, we work with the subsample of the population for which we have family background variables. Family background variables are only available when identification of parents is available in the central population register. This was not established until 1964, but was in part based on the census of 1960 (Skaug, 1968).

[4] This is also the measure on which the contributions to the National Insurance Scheme in Norway are calculated.

[5] Capital income is excluded from the analysis because it is difficult to get reliable data on capital income in Norway. The data on capital income are of low quality in Norway primarily because most people have their wealth invested in the house where they live. A large fraction of capital income is thus the gain from owning your own house and this gain is difficult to calculate as the value of houses is poorly registered. Furthermore, in order for capital income to add to the understanding of unfairness we would need to partition the set of capital income determinants into factors for which an individual *is*, and *is not*, responsible. It could, for example, be argued that people should be held responsible for how much they save, but not be held responsible for their inheritance. Information about the determinants of capital income is, however, not contained in any data set that we are aware of.

Table 2
Composition by gender: years of education, proportion in public sector, and hours worked.

	Male		Female	
	1986	2005	1986	2005
Mean years of education	11.21	12.47	10.48	12.45
Proportion in public sector	0.238	0.224	0.508	0.570
Proportion working less than 20 h/week	0.016	0.042	0.249	0.169
Proportion working 20–29 h/week	0.026	0.022	0.264	0.176
Proportion working more than 30 h/week	0.959	0.936	0.488	0.655

Hence, for the older cohorts, there is only limited information available on family background. For this reason we limit this subsample to those born in 1946 or later, who were mostly living with their parents in the 1960 census. For the 1986 sample, we are able to match the education of both parents and the county in which they lived in the 1960 census for 68% of these cohorts (332,240 individuals), in the 2005 sample we can match these variables for 86% of the cohorts (1,153,781) individuals.

For another subsample, we are able to use a direct IQ measure obtained from military records. All male, and some female, citizens undergo a medical examination at 17–18 years, intended to determine their fitness for military service. As part of this examination, there is a classification of their ability score in Standard Nine (stanine) units, which is a method of standardizing raw scores into a nine point standard scale with a normal distribution, a mean of 5, and a standard deviation of 2. This data is available for persons being examined from 1968 onwards (see also Black et al., 2010). For the 1986 sample, we are able to match the full set of family background and the IQ score for 44% of the male cohorts born after 1946 (111,533 individuals), in the 2005 sample we can match everything for 72% of the male cohorts (497,222 individuals).

We do not have reliable data on post-tax income that we can apply in our analysis. However, by using a model that captures the main features of the Norwegian income tax system in 1986 and 2005, we are able to impute individual post-tax incomes from the pre-tax data.[6]

4.2. The labor earnings equation

Table 1 presents some descriptive statistics on education and labor earnings, where the main observation is that there are no major structural changes between 1986 and 2005. The most important change in the period is captured in Table 2, which shows the composition of hours worked, the proportion working in the public sector and years of education, all broken down by gender. We observe two main developments that will be important for the later analysis, both related to females' economic situation. Females have catched up with males in years of education and a substantially larger proportion of females work more than 30 h in 2005 than in 1986.

We use a linear model of the logarithm of labor earnings,

$$\log y_i = \boldsymbol{\beta} \boldsymbol{x}_i^R + \boldsymbol{\gamma} \boldsymbol{x}_i^{NR} + \varepsilon_i, \qquad (6)$$

where $\boldsymbol{x}_i^R$ are the explanatory variables for which i is to be held responsible, $\boldsymbol{x}_i^{NR}$ are the explanatory variables for which i is not held responsible, and ε_i captures unobservable factors. To focus on our main research question, we make the assumption that ε_i is independent of $\boldsymbol{x}_i = (\boldsymbol{x}_i^R, \boldsymbol{x}_i^{NR})$.

Table 3 shows the regression results for 1986 and 2005. We observe that the estimated effect of working more than 30 h slightly decreases during the period, whereas the return for the group working between 20 and 30 h slightly increases. In 2005, working more than 30 h almost doubled the estimated labor earnings compared to working less than 20 h.

Table 3
Labor earnings equations.

	1986	2005
Hours worked: 20–29	0.374	0.312
	(0.00185)	(0.00163)
Hours worked: >30	0.728	0.622
	(0.00162)	(0.00126)
Years of education	0.0654	0.0653
	(0.000249)	(0.000172)
Employed in public sector	0.00894	−0.108
	(0.00112)	(0.000908)
Age: 40–49	0.100	0.122
	(0.00108)	(0.000879)
Age: 50–59	0.0517	0.131
	(0.00122)	(0.000921)
Female	−0.392	−0.252
	(0.00122)	(0.000899)
Humanities and arts†	−0.172	−0.124
	(0.00286)	(0.00195)
Teacher training and pedagogy†	−0.139	−0.103
	(0.00259)	(0.00187)
Social sciences and law†	−0.108	−0.0747
	(0.00446)	(0.00251)
Business and administration†	−0.00984	0.0284
	(0.00157)	(0.00129)
Natural sciences, vocational,	−0.0523	−0.0158
and technical†	(0.00161)	(0.00120)
Health, welfare, sport†	0.0201	0.0411
	(0.00196)	(0.00143)
Primary industries†	−0.158	−0.121
	(0.00402)	(0.00296)
Transport, security,	0.00489	0.0152
other services†	(0.00259)	(0.00200)
Unspecified†	−0.223	−0.229
	(0.0109)	(0.00518)
Constant	11.36	11.62
	(0.00402)	(0.0334)
Observations	994,296	1,335,078
R^2	0.481	0.401

Note: The table shows the coefficients estimated for the labor earnings equation given in Eq. (6). The dependent variable is individual labor earnings. Hours worked is a dummy coefficient for hours worked, and the omitted category is working less than 20 h. Individuals are only included if they are between 30 and 60 years of age. They are divided into three age categories, with less than 40 years old being the omitted category. Type of education is classified according to the person's final degree, where the different categories are marked with †. The omitted category is "General subjects", which indicates that the person's final degree has no specialized or tertiary-level degree. Persons with no registered classification, such as many immigrants, are indicated as "Unspecified". Nineteen county dummies are also included in the regression mode, but not reported in this table. Standard errors are reported in parenthesis.

The estimated coefficients on years of education are in line with previous studies, both on international and Norwegian data (Belzil, 2007; Haegeland et al., 1999). We also observe that both these estimates and the estimates on the types of education are almost constant during the period. In contrast, there is a substantial decrease in the estimated effect of gender. All else being equal, in 2005, the labor earnings of females are 78% of the labor earnings of men, while in 1986 these earnings are 68% of those of men. Keeping in mind that most females work in the public sector, as shown in Table 2, it is also interesting to observe that being employed in the public sector has a substantial negative effect on labor earnings in 2005, even after controlling for types of education, while there was no such effect present in the estimates for 1986. Finally, we observe that there is an increase in the estimated effect of age.

5. Measuring unfairness

Have the pre-tax and post-tax income distributions in Norway become more or less fair in the period 1986 to 2005? To address this

[6] Details of the tax model can be found on http://thomas.nhh.no/stata/norsk_skatt.html.

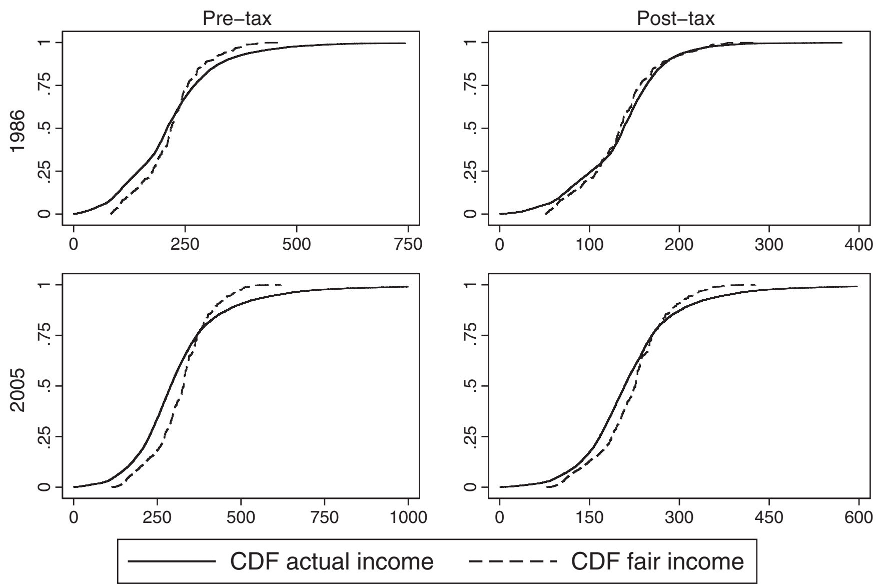

Fig. 2. The distribution of actual and fair incomes in 1986 and 2005. The figure displays the cumulative distribution functions (CDF) for actual income and fair income (in thousand NOK). The estimates of fair income are based on the responsibility set containing hours worked, years of education, sector (public versus private), and county of residence.

question we estimate the fair income distribution, and then use it to measure the development of the unfairness Gini in this period.

5.1. The fair income distribution

Simple algebraic substitution using Eqs. (5) and (6) provides the following expression of the estimated fair income for each individual[7]:

$$z_i^{GPP} = \frac{\exp\left(\boldsymbol{\beta x}_i^R\right)}{\sum_j \exp\left(\boldsymbol{\beta x}_j^R\right)} \sum_j y_j. \tag{7}$$

Individuals with different responsibility vectors may obviously have different fair incomes. To illustrate, it follows from the estimated labor earnings equation that the highest fair income in 2005 was close to five times as high as the lowest fair income. Overall, fair inequality, measured as the difference between the fair income distribution and perfect equality, decreased slightly over the period. The standard Gini for the fair income distribution fell from 0.176 in 1986 to 0.149 in 2005.

Differences in hours worked justify much of the fair inequality, but other responsibility factors also play an important role. The labor earnings estimates for 2005 show that it may be fair to give one person two and a half times more income than another who worked the same number of hours if they differ maximally with respect to the other responsibility factors.

Fig. 2 shows the cumulative distribution functions of fair incomes and pre-tax and post-tax incomes. As the cumulative distributions of fair income are steeper, less skewed, and concentrated in a smaller interval than the cumulative distributions for actual incomes, we know that the pre-tax and post-tax income distributions are less equal than the fair income distribution.

The general impression from a comparison of the cumulative distribution functions is that the distance between actual incomes, both pre-tax and post-tax, and fair incomes has increased from 1986 to 2005. We also note that the post-tax incomes are closer than the pre-tax incomes to the fair income distribution, particularly so in 1986.

However, cumulative distributions are not ordered on individuals, which is essential when fair incomes are individual-specific. By way of illustration, consider a situation where there is no unfairness. In this case, the cumulative distribution functions and labor earnings would completely overlap. But suppose now that we permute the labor earnings distribution. This would leave the cumulative distribution function for labor earnings unchanged, but might induce a substantial amount of unfairness.

Therefore we present the maps shown in Fig. 3, to show the extent to which each individual's pre-tax and post-tax incomes deviate from his fair income in 1986 and 2005. If all individuals received their fair income, then all observations should be on the diagonal line. All observations above the diagonal reflect individuals with a fair income higher than their actual income, whereas all observations below the diagonal represent individuals receiving an actual income higher than their fair income. The darkest area indicates the highest density of people. Comparing 1986 and 2005, we observe, for both pre-tax and post-tax incomes, that there are more observations further away from the diagonal in 2005, which indicates an increase in the unfairness of the income distribution in Norway during this period. We also observe that the post-tax maps, both in 1986 and 2005, are closer to the diagonal than the pre-tax maps, which indicate that the tax system in Norway contributes to decreasing unfairness.

5.2. Pre-tax income

The impression that there has been an increase in pre-tax income unfairness is confirmed by Fig. 4, which shows that there has been an increase in the unfairness Gini from 0.204 in 1986 to 0.220 in 2005. In contrast, the standard Gini for the pre-tax income distribution decreased in the period from 0.270 to 0.262. Hence, even though the changes are not large, we observe that the unfairness Gini provides a qualitatively different conclusion than the standard approach for the development of the pre-tax income distribution in Norway during this period.

There are two trends that explain most of this development. First, in line with what has been observed for a number of other countries in

[7] Let $C_i = \exp(\boldsymbol{\beta x}_i^R)$ and $D_i = \exp(\boldsymbol{\gamma x}_i^{NR} + \varepsilon_i)$, so $f(\boldsymbol{x}_i, \varepsilon_i) = C_i D_i$. Then

$$g\left(\boldsymbol{x}_i^R\right) / \sum_j g\left(\boldsymbol{x}_j^R\right) = n^{-1} \sum_j C_i D_j / \left(\sum_h n^{-1} \sum_j C_h D_j\right) = C_i \sum_j D_j / \sum_h C_h \sum_j D_j.$$

Since $\sum_j D_j$ is a constant, this simplifies to $C_i / \sum_h C_h$.

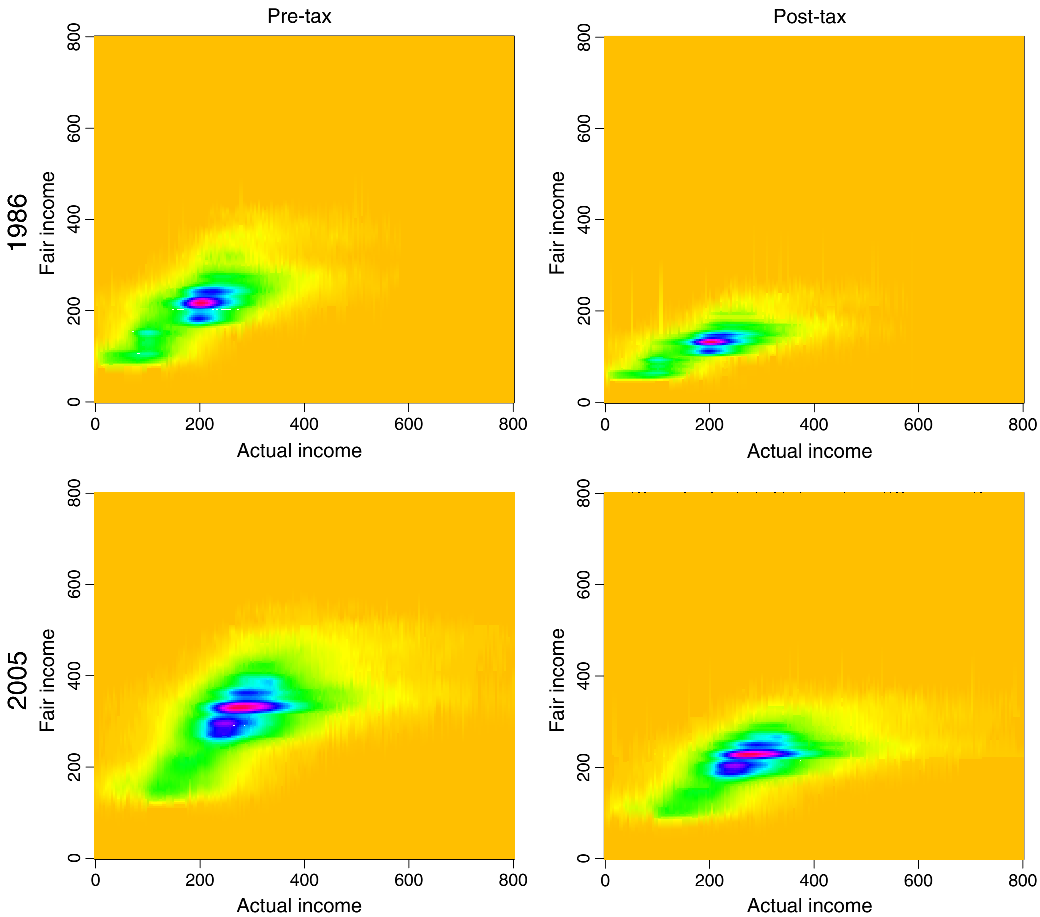

Fig. 3. The joint distribution of fair income and actual income in 1986 and 2005. The figures comprise a map that relates each individual's actual income to his or her fair income (in thousand NOK). The estimates of fair income are based on the responsibility set containing hours worked, years of education, sector (public versus private), and county of residence.

recent decades (Atkinson et al., 2011), there has been an increase in top labor incomes in Norway; the pre-tax income share of the top percentile increased from 3.41% in 1986 to 4.87% in 2005. The concentration of income at the top of the distribution increases both the standard Gini and the unfairness Gini, and can, in fact, account for almost all of the increase in the unfairness Gini. If we scale down the incomes of the top percentile in 2005 so that the share of total income for this group is the same as in 1986, the unfairness Gini drops from

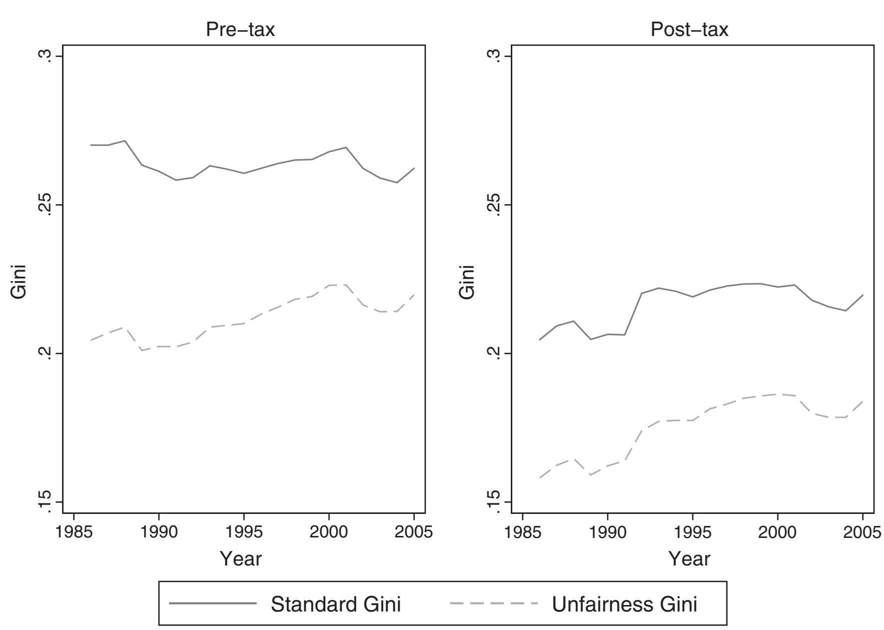

Fig. 4. Unfairness and inequality over time. The figure shows the development of the standard Gini and the unfairness Gini in the period 1986–2005. The estimates of fair income are based on the responsibility set containing hours worked, years of education, sector (public versus private), and county of residence.

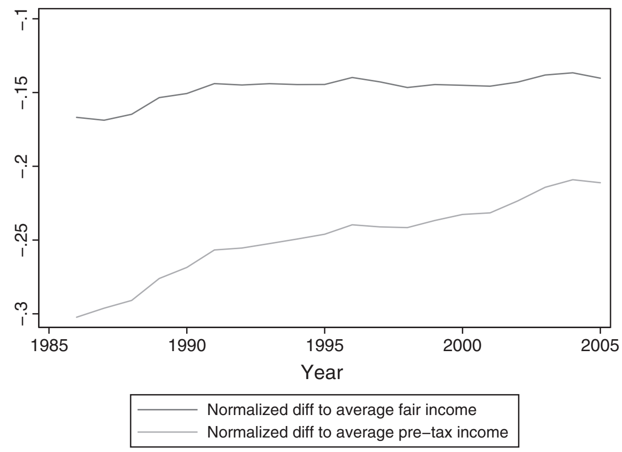

Fig. 5. The development of fair income and pre-tax income for females. The graph shows the normalized difference to average fair income, $\left(\overline{y}^f - \overline{z}^f\right) / \overline{y}$, and the normalized difference to average pre-tax income, $\left(\overline{y}^f - \overline{y}\right) / \overline{y}$, where the bars denote yearly averages and the superscript "*f*" denotes that the average is calculated for females only. The estimates of fair income are based on the responsibility set containing hours worked, years of education, sector (public versus private), and county of residence.

0.220 to 0.207, and, similarly, the standard Gini drops from 0.262 to 0.251.[8]

Second, we observed important changes in the situation of females in the period (Table 2). There is a sharp increase in the share of females working more than 30 h (from 48.8% to 65.5%) and also in females' level of education (from 10.47 years to 12.45 years). As a result, as shown in Fig. 5, the average pre-tax income of females is much closer to the average pre-tax income in society in 2005 than in 1986. In contrast, as can also be seen from Fig. 5, these changes do not bring females much closer to their fair income, since the increase in working hours and education also translates into an increase in females' fair income of almost the same size as their increase in pre-tax income. Thus, the increased role of females in the labor market impacts the standard Gini and the unfairness Gini differently. It causes a substantial decrease in the standard Gini that contributes to outweigh the effect of the increase in top labor incomes, whereas it has almost no impact on the unfairness Gini. As a result, the development of the two measures diverges, where we observe an increase in the unfairness Gini and a reduction in the standard Gini.

5.3. Post-tax income

We now turn to a study of unfairness in the post-tax income distribution, which is interesting on two accounts. First, in welfare studies, we are interested in the final outcome for individuals. Second, by comparing pre-tax and post-tax unfairness, we can gain insight into the extent to which the Norwegian tax system contributes to reducing unfairness.

Fig. 4 shows that there has been an even greater increase in post-tax unfairness than in pre-tax unfairness in the period; the unfairness Gini has increased with 16.5% for the post-tax income distribution (from 0.158 to 0.194), compared to an increase of 7.8% for the pre-tax income distribution. For the post-tax income distribution, we also observe an increase in the standard Gini of 6.8% (from 0.205 to 0.219).

Does the Norwegian income tax system contribute to reduced unfairness? A progressive tax system may have two opposing effects on unfairness. First, it may reduce unfair inequalities between individuals who are identical with respect to their responsibility factors. Second, it may eliminate fair inequalities between individuals who differ with respect to their responsibility factors. The first effect contributes to reduced unfairness, whereas the second effect contributes to increased unfairness.

We observe from Fig. 4 that the overall effect of the Norwegian income tax system is a reduction in unfairness both in 1986 and 2005. But the effect is larger in 1986 than in 2005; the tax system reduces the unfairness Gini with 22.6% (from 0.204 to 0.158) in 1986 and with 16.6% in 2005 (from 0.220 to 0.184). Hence, the tax reforms that have taken place in Norway between 1986 and 2005 seem to have made the tax system less capable of reducing overall unfairness in society.

6. Relocating the responsibility cut

So far, we have assumed that people are held responsible for the hours worked, years of education, sector (public versus private) and county of residence. However, this choice is clearly controversial, and we now discuss the implications of alternative specifications of the responsibility set.

Table 4 shows the development of the unfairness Gini for different responsibility sets. We observe that for pre-tax income, there is a striking difference between the standard approach and any responsibility-sensitive approach only including observable factors in the responsibility set. When comparing the data for 1986 and 2005, we observe a 3% decrease in the pre-tax unfairness Gini when the responsibility set is empty (the standard approach), whereas for all other specifications of the responsibility set the pre-tax unfairness Gini increases by about 6–12% in this period. The increase in unfairness is particularly large when people only are held responsible for the number of hours worked and years of education.

Our finding for the post-tax income distribution is also robust to any responsibility-sensitive approach only including observable factors in the responsibility set. In each case, the increase in post-tax unfairness from 1986 to 2005 is larger, from 17 to 21%, than the increase in the standard Gini of about 7%.

We also consider a responsibility set that includes the non-observable factors (last row in Table 4). Not surprisingly, this contributes to a substantial reduction in measured unfairness. We observe that the inclusion of the non-observable factors implies that both pre-tax and post-tax unfairness have decreased substantially in Norway, the unfairness Gini drops by almost 40% and 30%, respectively. This is mainly explained by the fact that unobservable variables are much more important in the labor earnings equation in 2005 than in 1986 (see the decrease in the R^2-value in Table 3).

Some may find it curious that we have considered the possibility of including age in the responsibility set, given that this is clearly a factor that is beyond individual control. However, the inclusion of this factor can be justified on pragmatic grounds, and serves as an approximation of the view that our concerns should be equality in expected lifetime labor earnings. This was a primary argument in the early literature on fair income (Paglin, 1975), and it is still present in public debate. However, in our analysis, this is not a fundamental issue, since the unfairness Gini only changes slightly when including age in the responsibility set.

Table 4
Unfairness Gini for different responsibility cuts.

Responsibility set	$\mathbf{G}^u$ (pre-tax)		$\mathbf{G}^u$ (post-tax)	
	1986	2005	1986	2005
∅ (standard Gini)	0.270	0.262	0.205	0.219
$\{H\}$	0.223	0.235	0.159	0.192
$\{H, E\}$	0.206	0.229	0.158	0.192
$\{H, E, P\}$	0.206	0.221	0.157	0.184
$\{H, E, P, D\}$	0.204	0.220	0.158	0.184
$\{H, E, P, D, F\}$	0.201	0.217	0.153	0.181
$\{H, E, P, D, F, A\}$	0.200	0.214	0.152	0.178
$\{H, E, P, D, F, A, \varepsilon\}$	0.120	0.076	0.098	0.069

Note: We consider the possibility of including hours worked (H), years of education (E), working in the public sector (P), county of residence (D), field of education (F), age (A) and the unobservable factors (ε) in the responsibility set. The responsibility set indicates which variables are considered to be part of $\boldsymbol{x}^R$, with the remaining being part of $\boldsymbol{x}^{NR}$.

[8] The incomes in the top percentile are scaled down according to the following formula, $y_i' = y_{99} + \alpha(y_i - y_{99})$, where y_{99} is the highest income in the 99th percentile in 2005.

It is interesting to note that the unfairness Gini is not necessarily decreasing monotonically as the responsibility set expands. For the post-tax distribution in 1986, we observe an increase in the unfairness Gini when we add the county of residence to the responsibility set. This illustrates the general point that deviation from the fair distribution may increase as we hold people responsible for more factors (Cappelen and Tungodden, 2006); it may even be the case that a responsibility-sensitive framework reports more unfairness than the standard approach of no responsibility, but this is not the case in our analysis.

7. Robustness analysis

In this section, we show that our main findings for the pre-tax income distribution are robust to the core elements in the framework, where our focus is on responsibility sets which only include observable factors.[9]

7.1. Innate ability and family background

A fundamental concern in the application of responsibility-sensitive theories of justice is that the responsibility factors might be affected by non-responsibility factors (see for example Betts and Roemer, 2007). In our study, one may question whether hours worked and years of education are affected by non-responsible factors such as family background and innate ability.

Fortunately, we have data that allows us to study this issue in more detail. We start by considering how controlling for family background might affect our analysis. For a subsample we have information about family background, and thus we can run regressions of hours worked and years of education on these variables. These regressions show that, for this subsample, family background correlates with years of education, but not with hours worked. To control for family background when holding individuals responsible for years of education, we construct a new variable given by the difference between the actual years of education and the predicted years of education based on family background. Panel A in Table 5 reports how our estimates of the pre-tax unfairness Gini for this subgroup are affected when years of education is replaced by this new variable in the responsibility set. We observe that the pre-tax unfairness Gini is lower when controlling for family background. The effect, however, is rather small and almost identical for 1986 and 2005; for our main specification of the responsibility set, the pre-tax unfairness Gini decreases with 0.01 in both years.

We are also able to study how innate ability might affect our estimates. For a subsample of males we also have access to a direct measure of cognitive skills from military records. Again, for this subsample, the non-responsibility factors turn out to be of importance for years of education, but not for hours worked. However, as reported in Panel B in Table 5, there is only a very minor change in our estimates of unfairness when controlling both for innate ability and family background on years of education.

Table 5
Pre-tax unfairness Gini corrected for family background and innate ability.

Responsibility set	Baseline		With corrections	
	1986	2005	1986	2005
A. Family background				
∅ (standard Gini)	0.260	0.260	0.260	0.260
$\{H\}$	0.218	0.233	0.218	0.233
$\{H,E\}$	0.217	0.233	0.208	0.229
$\{H,E,P\}$	0.218	0.231	0.207	0.221
$\{H,E,P,D\}$	0.215	0.229	0.205	0.219
$\{H,E,P,D,F\}$	0.215	0.231	0.201	0.218
$\{H,E,P,D,F,A\}$	0.215	0.230	0.200	0.217
B. Innate ability (IQ score) and family background				
∅ (standard Gini)	0.181	0.241	0.181	0.241
$\{H\}$	0.179	0.236	0.179	0.236
$\{H,E\}$	0.179	0.236	0.177	0.235
$\{H,E,P\}$	0.180	0.237	0.176	0.232
$\{H,E,P,D\}$	0.173	0.233	0.171	0.229
$\{H,E,P,D,F\}$	0.169	0.230	0.170	0.228
$\{H,E,P,D,F,A\}$	0.169	0.229	0.170	0.225

Note: The estimates with corrections are based on replacing years of education with a variable that captures the difference between the actual years of education and the predicted years of education. In Panel A, the prediction is based on family background (years of education of mother and father, interaction of these, and dummies for the childhood county), in Panel B it also controls for innate ability. Panel A reports results for the subsample for which we have family background variables; Panel B reports results for the subsample for which we have both family background variables and IQ-score.

7.2. Unfairness Lorenz dominance

So far, our analysis has relied on the unfairness Gini, and thus it is interesting to study the extent to which other measures of unfairness support the same conclusion. Fig. 6 shows the unfairness Lorenz curves for 1986 and 2005 for our main specification of the responsibility set, and we observe that there is no unfairness Lorenz dominance. Both the pre-tax and post-tax incomes of the most unfairly treated group are slightly closer to the fair income of this group in 2005 than it is in 1986, whereas the opposite is the case for the rest of the population. Moreover, we observe that there is a much greater difference between the two unfairness Lorenz curves for the upper tail of the distribution. Using all other non-empty responsibility sets (only including observable variables), we obtain similar patterns as in Fig. 6.[10] In each case, both for pre-tax and post-tax incomes, there is almost perfect overlap for the most unfairly treated group, and a marked difference for the rest of the population. Given the ordering of the responsibility sets $\{H\}$ to $\{H,E,P,D,F,A\}$ in Table 4, the single crossing occurs at the percentiles of 40.0, 20.8, 37.2, 37.0, 33.6, and 38.2 for pre-tax income and of 27.8, 12.8, 23.8, 24.6, 17.4, and 21.2 for post-tax income.

The lack of unfairness Lorenz dominance implies that there exist other unfairness measures satisfying the basic conditions introduced in Section 2, that would go against the conclusion of the unfairness Gini. However, given that the unfairness Lorenz curves almost overlap for the most unfairly treated, such measures would have to assign very high priority to the lower tail of the unfairness distribution. We doubt that there is political support for such priority to the most unfairly treated, and thus for practical purposes we tend to conclude that our finding of increasing pre-tax and post-tax unfairness in Norway is robust to the choice of unfairness measure.

It is also interesting to compare the unfairness Lorenz curves to the standard Lorenz curves, i.e. unfairness Lorenz curves with an empty responsibility set. The standard Lorenz curves, which are shown in Fig. 6, also have a single crossing, but they have a more marked difference for the most unfairly treated group. Consequently, the results derived on the basis of the standard Gini are less robust than our findings for the unfairness Gini.

7.3. Alternative responsibility-sensitive fairness principles

The generalized proportionality principle provides, in our view, an attractive formulation of the idea that inequalities due to non-responsibility factors should be eliminated, while inequalities due to responsibility factors should be accepted. However, other interesting responsibility-sensitive fairness principles have been proposed in the literature, and in this section, we study whether three alternatives, two versions of the egalitarian equivalent fairness principle and one version of the conditionally egalitarian fairness principle, give different results.

[9] Our findings for the post-tax distribution are also robust to the same elements, and the detailed analysis is available upon request from the authors.

[10] The unfairness Lorenz curves for the other non-empty responsibility sets are available from the authors upon request.

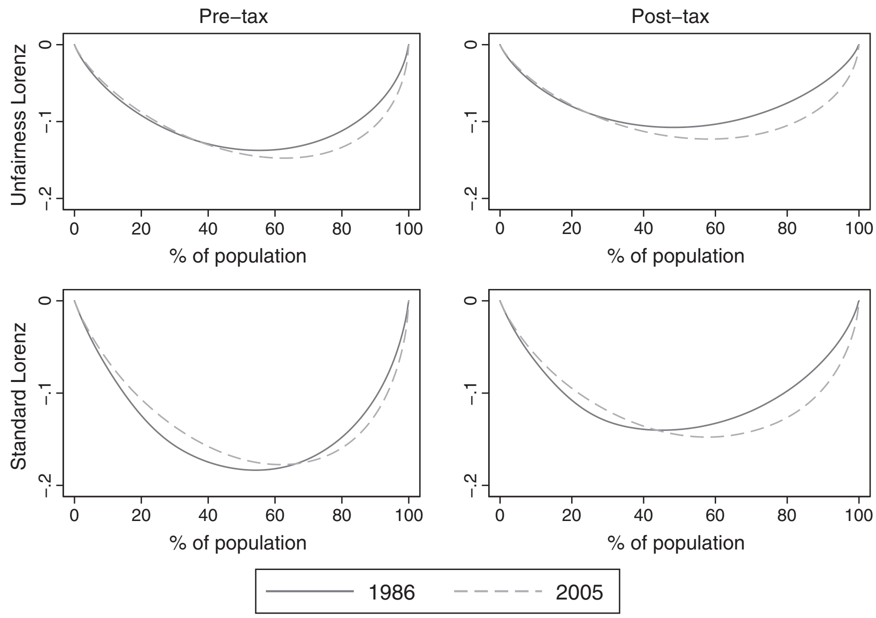

Fig. 6. Lorenz curves for 1986 and 2005. The estimates of fair income are based on the responsibility set containing hours worked, years of education, sector (public versus private), and county of residence.

According to one version of the egalitarian equivalent fairness principle (Bossert and Fleurbaey, 1996), the fair income of individual i is given by

$$z_i^{EE} = f\left(\boldsymbol{x}_i^R, \tilde{\boldsymbol{x}}^{NR}\right) + C^{EE}, \tag{8}$$

where $\tilde{\boldsymbol{x}}^{NR}$ is a reference non-responsibility vector, and C^{EE} is a constant such that the average fair income is equal to the average pre-tax income. We focus in the empirical analysis on the case where each element in the reference non-responsibility vector is defined by the average value of the corresponding non-responsibility factor in society.

Within the egalitarian equivalent framework, an alternative formulation of the idea that people's fair income should depend on the average non-responsibility vector in society is given by

$$z_i^{EE*} = \frac{1}{n}\sum_j f\left(\boldsymbol{x}_i^R, \boldsymbol{x}_j^{NR}\right) + C^{EE*}, \tag{9}$$

where C^{EE*} is a constant that ensures that the average fair income is equal to the average pre-tax income.

Finally, another prominent responsibility-sensitive fairness principle is the conditional egalitarian fairness principle (Kolm, 1996), which defines the fair income of individual i as given by

$$z_i^{CE} = f\left(\boldsymbol{x}_i^R, \boldsymbol{x}_i^{NR}\right) - f\left(\tilde{\boldsymbol{x}}^R, \boldsymbol{x}_i^{NR}\right) + C^{CE}, \tag{10}$$

where $\tilde{\boldsymbol{x}}^R$ is the reference responsibility vector and C^{CE} is a constant equal to what would have been the average income in the economy if everyone had responsibility factors described by the reference responsibility vector. In the empirical analysis, we focus on the case where each element in the reference responsibility vector is the average value of the corresponding responsibility factor in society. Note that this approach is less egalitarian than the other fairness principles we consider; it only justifies eliminating inequalities arising from non-responsibility factors among individuals with a responsibility vector equal to the reference responsibility vector.

Fig. 7 shows the development of both the pre-tax and post-tax unfairness Gini for each of the four fairness principles. We observe that the four approaches report the same trend. The generalized proportionality fairness principle and the two formulations of the equalitarian equivalent principle have almost identical implications; in fact, the generalized proportionality principle and the alternative formulation of the egalitarian equivalent principle cannot be distinguished in the figure. The conditional egalitarian principle implies a lower overall level of unfairness, which reflects that it is less egalitarian than the other fairness principles, but still supports our main finding of increasing unfairness in the pre-tax and post-tax income distributions in Norway.

8. Concluding remarks

In this paper we have generalized the standard approach to inequality measurement, where we allow for different interpretations of what is a fair income distribution. Within the generalized framework, we have estimated the fair income distribution in Norway from 1986 to 2005, and measured the unfair (in)equality during the same period. Interestingly, when studying the pre-tax income distribution, we find that even though the standard Gini is lower in 2005 than in 1986, there is an overall increase in unfairness. This finding is robust to changes in the responsibility set (as long as we do not hold people responsible for unobservable factors) and to alternative formulations of the unfairness measure and the underlying responsibility-sensitive fairness principle. We furthermore find that the Norwegian tax system contributes to a reduction in overall unfairness, but less so in 2005. Consequently, the increase in post-tax unfairness is even greater than in the pre-tax income distribution.

We believe that our findings illustrate the importance of incorporating the distinction between fair and unfair inequalities in empirical inequality studies, and we think that the approach proposed in this paper provides one promising framework for such analyses. A challenge to this approach, however, is that non-responsibility factors, such as innate ability and family background, may affect responsibility variables like hours worked and education, and thus it is important, as we do in our analysis, to control for such relationships. An interesting alternative approach would be to apply the theoretical framework of Fleurbaey and Maniquet (2006), who focus more directly on holding people responsible for their preferences. But this approach requires estimation of the distribution of individual preferences in the population, which is a difficult task given the standard types of data available.

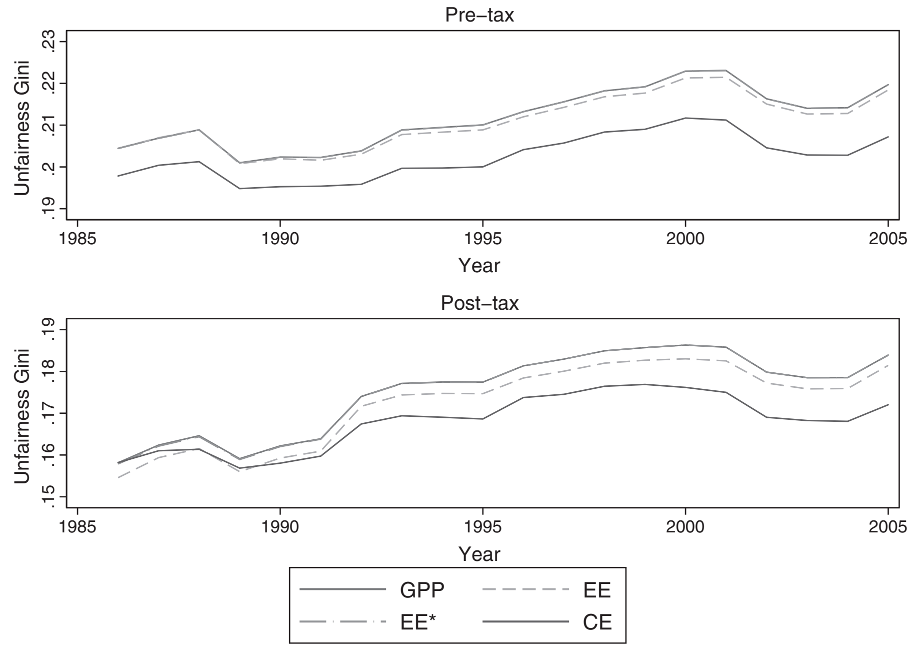

Fig. 7. Unfairness Gini for different fairness principles. The figure shows the unfairness Gini, both pre-tax and post-tax, for four different fairness principles: the generalized proportionality principle (*GPP*), the two versions of the egalitarian equivalent principle (*EE* and *EE*), and the conditional egalitarian principle (*CE*). The two formulations of the egalitarian equivalent principle cannot be distinguished in the panels. In the calculation of the post-tax unfairness Gini, we have scaled the fair incomes such that the sum of fair incomes is equal to the sum of post-tax incomes.

Appendix A. Proof of observation

Proof. Consider any $\widehat{\boldsymbol{A}},\widehat{\boldsymbol{B}}\in\Xi$, where $\widehat{\boldsymbol{A}}\ \mathbf{LD}^u\ \widehat{\boldsymbol{B}}$. We will now show that $\widehat{\boldsymbol{A}}\prec\widehat{\boldsymbol{B}}$ for any partial ordering satisfying the conditions stated in the observation.

(i) Consider $\boldsymbol{A},\tilde{\boldsymbol{B}}\in\Xi$, where $\left(y_i^{\boldsymbol{A}},z_i^{\boldsymbol{A}}\right)=\left(y_{i(\widehat{\boldsymbol{A}})},z_{i(\widehat{\boldsymbol{A}})}\right)$ and $\left(y_i^{\tilde{\boldsymbol{B}}},z_i^{\tilde{\boldsymbol{B}}}\right)=\left(y_{i(\widehat{\boldsymbol{B}})},z_{i(\widehat{\boldsymbol{B}})}\right)$ for all $i\in\boldsymbol{N}$. By Anonymity, $\boldsymbol{A}\sim\widehat{\boldsymbol{A}}$ and $\tilde{\boldsymbol{B}}\sim\widehat{\boldsymbol{B}}$. Moreover, consider $\boldsymbol{B}\in\Xi$, where $\boldsymbol{B}=\left(\mu\left(\widehat{\boldsymbol{A}}\right)/\mu\left(\widehat{\boldsymbol{B}}\right)\right)\tilde{\boldsymbol{B}}$. By Scale Invariance, $\boldsymbol{B}\sim\tilde{\boldsymbol{B}}$.

(ii) By (i), $\boldsymbol{A}\ \mathbf{LD}^u\ \boldsymbol{B}$. Hence, it follows that $u_{1(\boldsymbol{A})}\geq u_{1(\boldsymbol{B})}$. Consider now the following sequence:

$$
\begin{aligned}
\boldsymbol{A}_1 &= \boldsymbol{A},\\
\boldsymbol{A}_2 &= \Big[\Big(y_1^{\boldsymbol{A}}-\min\Big\{u_{1(\boldsymbol{A})}-u_{1(\boldsymbol{B})},\max\Big\{u_{2(\boldsymbol{B})}-u_{2(\boldsymbol{A})},0\Big\}\Big\},z_1^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_2^{\boldsymbol{A}}+\min\Big\{u_{1(\boldsymbol{A})}-u_{1(\boldsymbol{B})},\max\Big\{u_{2(\boldsymbol{B})}-u_{2(\boldsymbol{A})},0\Big\}\Big\},z_2^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_3^{\boldsymbol{A}},z_3^{\boldsymbol{A}}\Big),\ldots,\Big(y_n^{\boldsymbol{A}},z_n^{\boldsymbol{A}}\Big)\Big],\\
\boldsymbol{A}_3 &= \Big[\Big(y_1^{\boldsymbol{A}_2}-\min\Big\{u_{1(\boldsymbol{A}_2)}-u_{1(\boldsymbol{B})},\max\Big\{u_{3(\boldsymbol{B})}-u_{3(\boldsymbol{A})},0\Big\}\Big\},z_1^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_2^{\boldsymbol{A}_2},z_2^{\boldsymbol{A}}\Big),\Big(y_3^{\boldsymbol{A}}+\min\Big\{u_{1(\boldsymbol{A}_2)}-u_{1(\boldsymbol{B})},\max\Big\{u_{3(\boldsymbol{B})}-u_{3(\boldsymbol{A})},0\Big\}\Big\},z_3^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_4^{\boldsymbol{A}},z_4^{\boldsymbol{A}}\Big),\ldots,\Big(y_n^{\boldsymbol{A}},z_n^{\boldsymbol{A}}\Big)\Big],\\
&\ \vdots\\
\boldsymbol{A}_n &= \Big[\Big(y_1^{\boldsymbol{A}_{(n-1)}}-\min\Big\{u_{1\left(\boldsymbol{A}_{(n-1)}\right)}-u_{1(\boldsymbol{B})},\max\Big\{u_{n(\boldsymbol{B})}-u_{n(\boldsymbol{A})},0\Big\}\Big\},z_1^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_2^{\boldsymbol{A}_2},z_2^{\boldsymbol{A}}\Big),\ldots,\Big(y_{(n-1)}^{\boldsymbol{A}_{(n-1)}},z_{(n-1)}^{\boldsymbol{A}}\Big),\\
&\quad \Big(y_n^{\boldsymbol{A}}+\min\Big\{u_{1\left(\boldsymbol{A}_{(n-1)}\right)}-u_{1(\boldsymbol{B})},\max\Big\{u_{n(\boldsymbol{B})}-u_{n(\boldsymbol{A})},0\Big\}\Big\},z_n^{\boldsymbol{A}}\Big)\Big].
\end{aligned}
$$

For all $i=1,\ldots,n-1$, $\boldsymbol{A}_i=\boldsymbol{A}_{i+1}$ or $\boldsymbol{A}_i\preceq\boldsymbol{A}_{i+1}$ according to Generalized Pigou–Dalton. If $u_{1(\boldsymbol{A})}=u_{1(\boldsymbol{B})}$, then $\boldsymbol{A}_n=\boldsymbol{A}$. Otherwise, by transitivity and Generalized Pigou–Dalton, $\boldsymbol{A}_n\succ\boldsymbol{A}$.

(iii) By (ii) and the fact that $\sum_i u_{i(\boldsymbol{A})}=0$, it follows that $u_{1(\boldsymbol{A}_n)}=u_{1(\boldsymbol{B})}$. Moreover, by the fact that $\boldsymbol{A}\ \mathbf{LD}^u\ \boldsymbol{B}$, it follows that $u_{2(\boldsymbol{A}_n)}\geq u_{2(\boldsymbol{B})}$. If $u_{2(\boldsymbol{A})}\geq u_{2(\boldsymbol{B})}$, then individual 2 has not received anything in the sequence in (ii). Hence, it follows that he is at least as badly off as persons $3,\ldots,n$. Alternatively, if $u_{2(\boldsymbol{A})}<u_{2(\boldsymbol{B})}$, then it follows from the fact that $\boldsymbol{A}\ \mathbf{LD}^u\ \boldsymbol{B}$ that $u_{2(\boldsymbol{A}_n)}=u_{2(\boldsymbol{B})}$. In any case, we may repeat exactly the same kind of sequence as in (ii), where we take from person 2 if $u_{2(\boldsymbol{A}_n)}>u_{2(\boldsymbol{B})}$ (otherwise nothing will happen in the sequence) and give to persons $3,\ldots,n$. As a consequence, we will establish $\boldsymbol{A}_n^2$, where $u_{2\left(\boldsymbol{A}_n^2\right)}=u_{2(\boldsymbol{B})}$ and $\boldsymbol{A}_n^2=\boldsymbol{A}_n$ or $\boldsymbol{A}_n^2\succ\boldsymbol{A}_n$. Similarly, we can do this for persons $3,\ldots,n$. In sum, we will establish $\boldsymbol{A}_n^n$, where $u_{i\left(\boldsymbol{A}_n^n\right)}=u_{i(\boldsymbol{B})}$ for all i and where, by transitivity and Generalized Pigou–Dalton, $\boldsymbol{A}_n^n\succ\boldsymbol{A}$.

(iv) By (iii) and Unfairism, it follows that $\boldsymbol{A}\prec\boldsymbol{A}_n^n\sim\boldsymbol{B}$. Hence, by transitivity and (i), we have that $\widehat{\boldsymbol{A}}\prec\widehat{\boldsymbol{B}}$. □

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