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# EDUCATIONAL PRODUCTION*

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Classroom education has public good aspects. The technology is such that when one student disrupts the class, learning is reduced for all other students. A disruption model of educational production is presented. It is shown that optimal class size is larger for better-behaved students, which helps explain why it is difficult to find class size effects in the data. Additionally, the role of discipline is analyzed and applied to differences in performance of Catholic and public schools. An empirical framework is discussed where the importance of sorting students, teacher quality, and other factors can be assessed.

There exists an enormous empirical literature on the relation of educational attainment to class size. Results in this literature vary from significant class size effects to no (or sometimes even perverse) class size effects.[1] The inability to find consistent class size effects is most perplexing. At some basic level, the failure to observe class size effects makes no sense because observed class

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1. See Hanushek [1998b] who finds little evidence that anything matters, including class size reductions. Coleman and Hoffer [1987] and Coleman, Kilgore, and Hoffer [1981] report that Catholic schools with large class sizes produce better students than public schools with smaller class sizes. Class size effects are documented by a number of authors. The literature goes very far back. For example, Blake [1954] summarized a literature where 35 studies found smaller class size was better, 18 found larger class size was better, and 32 were inconclusive. More recent are Hanushek [1998a], Hoxby [1998, 2000a], and Krueger [1998, 1999]. Angrist and Lavy [1999] find that class size matters for elementary school children in fourth and fifth grade in Israel. Lavy [1999], using an experimental design, finds that class size does not matter in OLS regressions, but does when political variables are used, exploiting a discontinuity structure.


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size is generally smaller than the entire number of students at any particular grade level. Why bear the expense of having four kindergarten classes of 30 rather than one class of 120 if class size truly does not matter? Furthermore, observed class size varies with age of the student. Preschool classes are smaller than large lecture classes for college students. How is this to be explained if class size is irrelevant?

Although there is a vast empirical literature on educational production and its determinants, there is a relatively small theoretical literature.[2] In what follows, a theory is presented that addresses the class size puzzle. The basic structure begins with the recognition that education in a classroom environment is a public good. But as with most public goods, classroom learning has congestion effects, which are negative externalities created when one student impedes the learning of all other classmates. There is empirical support for this proposition. Peer effects have long been recognized as crucial in education.[3] While hardly novel, to understand peer interaction effects, it is necessary to embed the spillovers in a framework where changing the size of a class or its composition has a cost. The primary cost takes the form of teacher salary and infrastructure.

The answer to the class size puzzle rests on the realization that class size is a choice variable and the optimal class size varies inversely with the attention span of the students. It is efficient to use fewer teachers and a higher student-teacher ratio when the students are better behaved. But an envelope theorem implies that actual educational output varies directly with the behavior of the student, despite the fact that fewer teacher inputs are used. An implication is that class size matters, but the observed relation of educational output to class size may be small or even positive.[4]

2. Some exceptions are Caucutt [1996] and Fernandez and Rogerson [1998]. The focus of these studies is somewhat different from the one here, but there is overlap in deriving sorting equilibria. Positive assortive mating, of the kind derived in these studies, goes back at least as far as Becker [1991]. Closest to the analysis contained herein is work by Brown and Sachs [1974].

3. There is a large literature here. An early empirical paper is Henderson, Mieskowski, and Sauvageau [1978].

4. Hanushek [1998b] reports that expenditures per student more than doubled between 1960 and 1990 at a time when there was no steady trend in test scores or other measures of performance. Other recent papers to examine the relation between expenditures and class size are Card and Krueger [1992] and Betts [1996]. There is evidence that competition both lowers costs and improves school performance. See Hoxby [1996, 1998], McMillan [1999], and Urquiola [1999].

The main purpose of the model presented here is to tie together a wide variety of facts and to integrate the literature on class size and student performance. A further goal is to provide a new empirical strategy for understanding student performance and the determinants of it. The goal is ambitious, but it is hoped that some progress can be made by emphasizing the role that behavior plays in the determination of class size. The primary implications of the theory are the following.

1. Optimal class size varies directly with student behavior and with the value of human capital and varies inversely with the cost of teachers. As a result, educational output per student can be lower in smaller classes.
2. The effect of reducing class size depends on the size of the class and the behavior of the students in it. Class size effects are larger for less well-behaved students.
3. Classes segregated by ability are the outcome of a private educational system and are efficient under a wide variety of circumstances.
4. The trade-off between discipline and class size is modeled and can be estimated empirically. Further, classroom etiquette and class size are determined simultaneously.
5. An exact function relates class size, student behavior, and educational output. The theory permits a new metric of school quality, and data currently exist that permit estimation of the relation and testing of the model.

## I. Model

The driving force is the idea that peer effects are important in classroom education. At some level the point is obvious. A classroom almost defines what is meant by a public good.[5] In the context of the public goods discussion, the cost of adding additional students can be thought of as congestion effects. In this setting, however, it is better to model the congestion effect more explicitly by thinking in terms of negative externalities that students may convey on one another. In classroom education, the

5. Heckman [1998, 1999] points out that learning is a lifetime affair and that the emphasis on formal schooling is misplaced, particularly when it is recognized that early success breeds later success. There is one major difference between formal schooling and the learning that occurs from infancy and continues after the cessation of school. Formal classroom schooling has public good attributes and externalities are key, whereas training given by a parent to his child or by an employer to his employee is essentially a private good.

ability of one student to get something out of a moment of class time depends on the behavior of others in the class. This is a clear application of the bad apple principle. If one child is misbehaving, the entire class suffers. Thus, let $p$ be the probability that any given student is not impeding his own or other's learning at any moment in time. Then, the probability that all students in a class of size $n$ are behaving is $p^n$ so that disruption occurs $1 - p^n$ of the time.[6]

The impediment may take a variety of forms. Student disruption provides a concrete example. Neither the student nor his classmates can learn much when the student is misbehaving, causing the teacher to allocate her time to him. Less nefarious, but equally costly is time taken by a student who asks a question to which all other students know the answer. One can think of $p$ as the proportion of time that a given student does not halt the public aspects of the classroom education process. Thus, the assumption made is that one child's disruption destroys the ability of all students (including himself) to learn at that moment.[7] It is expected that $p$ would be relatively high because even having $p =$ .98 in a class of 25 students results in disruption 40 percent of the time ($1 - .98^{25} = .40$). Disruptive behavior may be viewed as deviant behavior, but most students are capable of disrupting for at least some fraction, in this case, $1 - p$, of the time, especially when disruption is interpreted to mean asking a question to which others know the answer.

In contrast to negative externalities, students also provide public goods to one another. Although true, it is uninteresting to look to the range of class size values where adding students produces positive rather than negative externalities. Because increasing class size reduces cost per student, the profit-maximizing school will always increase class size to at least the point where additional students have negative effects on others. The optimum must be in the range where externalities are negative, and so the focus is on this part of the story throughout the paper.

So far, only technology has been discussed; the foregoing says

6. Actually, $1 - p$ is the probability that a given student initiates a disruption. It does not matter, given the technology postulated, that others may or may not follow. Furthermore, it could also be assumed that the individual who asks the question benefits from that time even if others do not. This will change the functional form only slightly as educational output per person rises from $p^n$ to $p^{n-1}$. All results remain qualitatively identical.

7. This technology is that of perfect complementarity. Kremer [1993] also presents a model using a perfect-complements-style technology.

nothing about optimality. To determine the schools' actions under varying conditions, let us begin by asking how much a student would pay to be in a class of size $n$. Suppose that the value of a unit of learning is given by $V$, which is determined by the market value human capital and the likelihood that a student is focusing on learning during the given instant.

To determine optimal class size, consider a school of $Z$ students with $m$ teachers and $m$ classes. Let the cost of a teacher and the rental value of the associated capital for her classroom be given by $W$. For now, $W$ is assumed to be independent of $p$ and other working conditions. Then a private school that wants to maximize profits can sell each moment at the school for $ZVp^n$, at a total cost of $Wm$. Maximization of profit would mean choosing $m$ so as to maximize

$$\text{Profit} = ZVp^{Z/m} - Wm \tag{1}$$

or equivalently,

$$\text{Profit per student} = Vp^n - W/n \tag{1a}$$

because each class has $n = Z/m$ students in it.

The first-order condition is then

$$-V\frac{Z^2}{m^2}p^{Z/m}\ln(p) - W = 0 \tag{2}$$

or using (1a),

$$Vp^n \ln(p) + W/n^2 = 0. \tag{2a}$$

Variations in $V$ can result either because the market for educated individuals relative to less-educated individuals changes, or because the amount of learning that takes place during an uninterrupted moment of schooling changes. However, as a modeling strategy, little emphasis is placed on variations in $V$. The goal is to attempt to provide as many testable predictions as possible by focusing on variations in $p$. Variations in teacher and student quality as well as other differences in the production function can always be thought to enter through $V$, but they are not the focus of the model.

Market equilibrium is achieved in two ways, given any existence of positive profits. Competitive entry of firms into the education industry drives up $W$ through demand pressure on wages in the teachers market. At the same time, the supply of educated graduates to the labor market drives down $V$. Equilibrium occurs when maximization of profit results in zero overall profit to the competitive supplier of education.

This is the basic model, and a number of implications can be derived from it.[8]

## II. Comparative Statics

Using equation (2), a proposition, proved in the Appendix, can be derived.

PROPOSITION 1. The optimal class size rises in teacher's wage, falls in the value of a unit of education, and most important, rises in the probability that students behave well. It is optimal to reduce class size when students are less well-behaved.[9]

The main purpose of the model is to provide a framework for discussing class size and how it varies with a number of factors.[10] To get a feel for this, consider an example. Normalize $V$ to 1. Then $W$, the price of a teacher's time, must be priced relative to $V$. In equilibrium, the price of teacher time relative to the productivity of student time must be sufficiently low to make the activity profitable. If it were not, private schools could not exist.

Suppose that the ratio of $W$ to $Vp^n$ is 5. The teacher's time is five times as valuable as what any one student gets out of the class. Then, if $Z = 100$ and $p = .99$ so that 99 percent of the time any given student is not causing enough disruption to interrupt learning in the classroom, the first-order condition (2) yields an optimum $m$ of 3.94, which gives a class size of 25 students.

This example makes clear why it is so difficult to find significant class size effects.[11] Increasing class size from 25 to 27 would reduce educational output per student by only about 2 percent.[12]

The more important point is that class size is a choice vari-

8. In a very fine unpublished paper, Brown and Sachs [1974] constructed a model with an educational production function that has features similar to those in this paper. First, schools maximize something, in their case, mean and minimum variance of test scores, by choosing resource allocations. They allow for there to be some public goods production of educational services.

9. The model is constructed in a way that ensures that class size is not a function of school size. However, Bedard, Brown, and Helland [1999] have found some school size effects.

10. The model could, in principle, be generalized to consider other kinds of expenditures as well. Focusing on class size is probably the place to start, particularly since Flyer and Rosen [1997] show that almost all of the rise in the cost of schooling over time is a result of reductions in class size.

11. Krueger [1999] is one of the few exceptions.

12. Akerhielm [1995] finds that the poorer-performing students are likely to

able. Because class size is a choice variable, researchers often observe small, or possibly even positive class size effects in cross-sectional data. The optimal number of teachers declines with $p$, which means that better-behaved students are in larger classes. The relation of $n$ to $p$ explains why kindergarten classes are smaller than college lectures. Furthermore, the fact that the optimum $n$ depends on $p$ and generally provides an interior solution explains why there are four kindergarten classes of 30, rather than one of 120. If $p$ were .97, learning would occur 40 percent of the time in a class of 30, but only 2½ percent of the time in a class of 120.

Although more disruptive students, who are themselves poorer learners, are found in smaller classes, the effect of reducing class size is not sufficient to overcome their deficiencies. Thus,

PROPOSITION 2. After optimal class-size adjustment, educational output per student is higher in the larger classes with better-behaved students than in the smaller classes with less well-behaved students. (Proved in the Appendix.)

There is substitution, but it is incomplete. Educational output in high $p$ classes is higher. If class size varies primarily because schools are adjusting class size in response to the behavior of the students, then the larger classes will have the better students and higher educational output, providing a positive observed relation between class size and educational output.[13] In the example above, when $p = .99$, optimal class size is 25, and educational output per student is .78. When $p$ falls to .98, the optimal class size is 19, and educational output per student is .68 because the effect of the lower value of $p$ swamps the effect of the reduced class size. This impairs the ability of the researcher to find improved educational output when class size is reduced.[14] The incongruity of educator statements and common sense with the failure to find class size effects is reconciled once it is recog-

end up in small classes. She also suggests that this may create bias in estimates of class-size effects.

13. Becker and Murphy [1992] show that optimal teacher-student ratio varies with the distance from the final product produced. This, too, provides a relation of equilibrium class size to technological parameters, although their focus is somewhat different from the one of this paper.

14. Olson and Ackerman [1999] find that wages are positively related to the pupil-teacher ratio of the schools that they attended. They attribute this to differences in teacher quality across districts.

nized that class-size, student characteristics, and educational output are related in a particular way.[15]

It is also clear why class-size effects are potentially quite important despite the inconsistency of the data. If a given group of students with a given value of $p$ were to be placed in a larger class, educational output would fall. Although the endogeneity of class size is well-understood, the point is not simply that poorer-quality students may be in smaller classes. When class size adjusts optimally, reductions in class size do not offset the effect of slower learning. A natural experiment that leaves $p$ constant and changes class size should induce the expected class-size effect. Even if large classes have more educational output, reducing the size of a given class would increase educational output further.[16] Angrist and Lavy [1999] and Krueger [1999] obtain this result, but Hoxby [2000a], who also uses a natural experiment approach, does not.[17]

The point is that even if class size effects are potentially important, in equilibrium, marginal changes in class size may have small effects on observed educational output. If large gains were available from lowering class size, then those changes would have been made.[18]

Since class size varies inversely with $W$, large class-size effects are most likely to be observed when the cost of teachers is low. Low teacher salaries imply low optimal class sizes. Reducing class size has a larger effect on educational output in small classes than in large ones. The empirical implication is that class size effects are most likely to be observed when teachers are relatively inexpensive. Preschool teachers are less expensive than professors, which generates small class size for preschoolers. Because of the nonlinearities in educational production function, class-size effects should be more important in preschool classes than they are at the college level.

15. Japan has high test scores and large class sizes. The finding is consistent with having a group of high $p$, well-behaved students.

16. Brown and Sachs [1974] also make this point.

17. Betts [1997] and Shkolnik and Betts [1999] provide evidence on class-size effects that vary, depending on what is held constant. The model in this paper provides implications for their data as well.

18. Krueger [1999] finds that the benefit of reducing class size is roughly equal to the costs.

*A. Behavior and Class Size*

What happens as $p$, the probability that a student is a nondisruptive learner, changes? There are two obvious applications, one relating to age and the other to underlying social behavior of a student body. Age is the most straightforward. Younger children have shorter attention spans than does the typical college student. In a class of kindergarten children, the probability that a child is behaving is lower than that in a college class.

Even within grade level, class size varies with topic. Some topics require more discussion and tend to have lower class size. If other student time were as valuable as teacher time, there would be no need to have small classes. It would not matter whether another student or the teacher was speaking. Because these effects are negative, at least on the margin, "air time" devoted to other students has negative effects on learning. These lower $p$ classes have smaller optimal class sizes, and lower educational output, other things the same.[19]

The implication is that if $V$, the value of a minute of first-grade schooling is the same as the value of a minute of college schooling, then the first-grade class should be smaller than the college class. Although not particularly surprising, it is a direct implication of a model that takes into account the cost of teacher time and trades this off against the gains from having a less frequently disrupted class.

Further, the marginal value of reducing class size is greater for low $p$ students than for high $p$ students. If special needs children have lower $p$ values than other children, then the implication is that special education classes should be smaller than regular education classes. Sufficient for this implication is that the value of a moment of education for special needs children is as high as it is for other children.

Angrist and Lavy [1999] and Krueger and Whitmore [1999] report that the class-size effects are more important for disadvantaged students than for others.[20] Suppose that disadvantaged students are also low $p$ students. Then, the proportionate in-

19. An early economics paper on class size by Summers and Wolfe [1977] found different effects for low achievers than for high achievers. Low achievers benefit from smaller class size, but the reverse is true for high achievers. Their results are somewhat consistent with this model because low achievers probably have higher values of $p$, where the reduction in size effects are greater.

20. Using the STAR data, Rouse [2000] also finds that smaller classes raise performance for inner city and minority students.

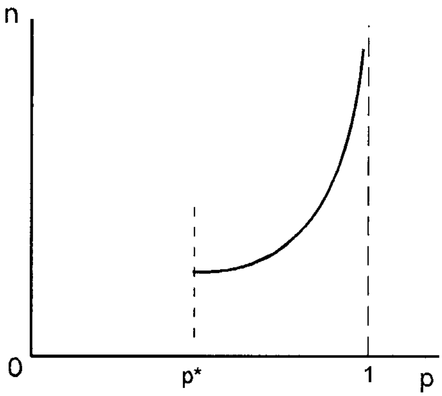


FIGURE I

crease in educational output associated with a decrease in class size from $nk$ to $n$ students is

$$(p^n - p^{nk})/p^{nk}.$$

Differentiate with respect to $p$ to obtain

$$d/dp = -np^{(n-nk-1)}(k-1) < 0$$

for $k > 1$. This implies that the effect of a class-size reduction is greater for low $p$ students than for high $p$ students.

### *B. Private Schools and Severe Behavior Problems*

Very poorly behaved children who have sufficiently low values of $p$ cannot be accommodated by a private school that must earn nonnegative profits. This is obvious from (1) since for $p = 0$, profits are negative for any positive value of $m$ and therefore, any finite value of $n$.

Figure I depicts the relation between the optimal $n$ and $p$. As $p$ declines, $n$ declines until $p$ reaches $p^*$, at which point it does not pay to supply any education. The firm provides no teachers and sets $m = 0$. Of course, no one would buy education of this sort in a private market. In the public sector, both schools and students might be forced to attempt to provide education, even to very low $p$ students. There a social planner who took only efficiency considerations into account would be induced to set $m$, the number of teachers per student, as close to zero as legally possible, creating large classes where there is little pretense of education. This situation might be thought of as a babysitting role of schools.

There exist private schools that cater to very low $p$ students, particularly those whose focus is special education. They exist primarily because society has decided that the value of educating these low $p$ individuals is higher than the private value of $V$ and are subsidized as a result. Subsidies are generally confined to younger low $p$ students. Implicitly, society imputes a high value to early education. But society is generally reluctant to subsidize older low $p$ students. College subsidies are rarely given to disruptive or slow-learning students. The pattern of subsidy is consistent with a view that there is little deviation between the private value of schooling and the social value of schooling for college students.

### *C. Differences in the Value of Education by Grade Level*

As already mentioned, the approach in this paper is to ignore differences in educational value across students as a way to explain observed class size, focusing instead on variations in behavior, as proxied by $p$. Still, it is useful to consider the effects of the value of education on class size. Proposition 1 states that an increase in $V$ implies a decrease in class size. Because elementary education is the foundation for all that follows, the value of knowledge acquired during a year of schooling might be expected to be highest in the earlier grades. ("Everything I need to know, I learned in kindergarten.") The fact that $p$ is lower for young children than for older ones coupled with the observation that wages for elementary school teachers are significantly lower than those for college professors implies that all of the factors push in the direction of smaller class size for younger children.

### *D. Special Programs*

Schools sometimes set up special programs and allocate slots in these programs on some kind of lottery or first-come-first-served basis. The programs often involve smaller class sizes and sometimes feature a different or extended curriculum. Unless the purpose of such programs is experimentation, it cannot be efficient to set up special programs with random student assignment. When lotteries or other nonattribute-based selection processes are used, the pool of selected students will have the same characteristics as those of the nonselected attributes so $p$ does not differ over students. Since (2) implies a unique optimum for $m$

and therefore $n$, deviating from the optimum by making some classes smaller and others larger cannot improve allocations. If a deviation in either direction were an improvement, it would be possible to choose every class with that alternative class size and do even better. Therefore, there are no asymmetrically efficient solutions when all students have the same $p$. Note that nothing in this discussion requires that the material taught in the different classes be the same. The same logic applies to new superior curricula. If it is optimal to provide the new curriculum to some group of the students, then it is optimal to provide it to all students because students are ex ante identical. Special classes allocated randomly fail the test of ex post fairness and are also inefficient. The best argument for asymmetric classes is that of experimentation.

## III. Sorting

To this point, it has been assumed that all students in a given class have the same behavior pattern. There are two questions that arise: first, is it efficient to integrate students, or should they be segregated into behavior-homogeneous classes? Second, does a market-based system of private education induce students to self-sort? In this section it is shown that segregation is efficient and that a market system does induce self-sorting.

### *A. Segregation by Type*

Suppose that there are two types of students. Let the $A$ group have a higher value of $p$ than the $B$ group so that $p_a > p_b$. While $A$ and $B$ need not refer to grades earned by such students, the interpretation is not an unnatural one.[21] A direct implication of Proposition 1 is that class size for $A$ students is larger than class size for $B$ students. It is now possible to state Proposition 3.

Proposition 3. Total output is maximized when students are segregated by type.

To prove this proposition, first assume that all classes are of

21. In the context of two different types, a question by a $B$ student might be viewed as disruption to $A$ students if all the $A$ students already know the answer and $B$ students do not.

size $n$. Suppose that the economy consists of $\alpha$ $A$ students and $1 - \alpha$ $B$ students. Then a school with matched classes[22] has output

(3) Output per student with segregated schools

$$= \alpha p_A^n + (1 - \alpha)p_B^n;$$

whereas output per student in an economy with integrated schools is

(4) Output per student with integrated schools $= p_A^{\alpha n} p_B^{(1-\alpha)n}$.

To show that it is always better to match than mix, it is merely necessary to show that the difference between the right-hand side of (3) and (4) is positive. The difference is

$$\text{diff} = \alpha p_A^n + (1 - \alpha)p_B^n - p_A^{\alpha n} p_B^{(1-\alpha)n}. \tag{5}$$

When $p_A = p_B$, diff $= 0$, as it must because then there is only one type, so mixing and matching is irrelevant. Next, differentiate (5) with respect to $p_A$ to obtain

$$\frac{\partial \text{diff}}{\partial p_A} = \alpha n p_A^{n-1}\left(1 - \frac{p_B^{(1-\alpha)n}}{p_A^{(1-\alpha)n}}\right) > 0$$

for $p_A > p_B$. Thus, the difference is zero for $p_A = p_B$ and becomes positive for $p_A > p_B$. Since $p_A > p_B$, school output is maximized by matching rather than mixing student types in classes.

Now, allow the choice of class size to differ. Define $n$, in (3), (4), and (5) to be the class size that is optimal when student types are mixed. Allowing segregation also allows schools to adjust class size optimally so as to allow different size classes for $A$'s and $B$'s. But segregation dominates even when all classes are constrained to be at the mixed optimum. Segregation must surely dominate if segregated classes can be of different sizes, which completes the derivation of Proposition 3.

Coleman and Hoffer [1981] and Coleman, Kilgore, and Hoffer [1987] report that performance is higher in private schools, and Catholic schools in particular, than in public schools.[23] One possibility is that Catholic schools expel the troublemakers, leaving a population of students who are easier to teach than those in the public schools. However, the facts are that expulsion rates are lower in the Catholic schools than they are in the private schools.

22. It is assumed that $\alpha$ and $m$ are such that proportions work out to guarantee an integer number of classes, each of which has $n$ students.

23. See also Evans and Schwab [1995], Neal [1997, 1998], and Sander [1997] for more recent papers.

It is possible that sorting occurs at admission. Given that public schools are free and that private schools cost, there is a positive difference in price associated with going to a private school.[24]

It is straightforward to show that $A$'s are willing to pay a higher price for admission to an all $A$ school than are $B$'s, although both are willing to pay a positive price. An $A$ receives value $p_A^n$ from an all $A$ school and $p_A^{\alpha n} p_B^{(1-\alpha)n}$ from an integrated school. ($V$ is normalized to 1.) A $B$ receives value $p_A^{n-1} p_B$ from an all $A$ school and $p_A^{\alpha n} p_B^{(1-\alpha)n}$ from an integrated school. The difference between what an $A$ will pay and what a $B$ will pay to move from the integrated school to an all $A$ school is then

$$p_A^n - p_A^{n-1} p_B,$$

which is positive. Thus, $A$'s are more likely to pay private school tuition to get into an all $A$ school than are $B$'s, given that the alternative is free public school.[25] The students who will pay the most to go to a private school are students with higher values of $p$, which is consistent with selection effects on Catholic schools working through admission, rather than expulsion.[26]

The intuition is this: $B$'s benefit from being around $A$'s, but $A$'s also benefit from being around other $A$'s. If there is a group of $n - 1$ $A$ students who will let one more student into the class, all of the current classmates prefer to admit an $A$. Furthermore, an outside $A$ gets more from entering an all $A$ class than does an

24. If all schools were private and competitive, $A$ schools would be less costly than $B$ schools, since the latter optimally use more teachers per student.

25. In equilibrium, if all $A$'s go to private schools, then the public schools consist only of $B$'s. It remains true, however, that $A$'s will pay more than $B$'s to be in an all $A$ school than in an all $B$ school. The difference between what an $A$ will pay and what a $B$ will pay is

$$[Vp_A^n - Vp_B^{n-1} p_A] - [Vp_A^{n-1} p_B - Vp_B^n],$$

which is positive for $p_A > p_B$.
Further, all $B$ private schools charge more than all $A$ private schools because optimal class size is smaller in the all $B$ school, raising costs. If public schools are already exclusively $B$, private $B$ schools could not compete unless they differed on other dimensions of quality.

Rothschild and White [1995] show that if type-specific prices can be charged, then a competitive equilibrium results in optimality. Becker and Murphy [2000] discuss market-induced sorting in the presence of externalities. Without a sufficient number of prices, there is inefficient allocation. In this context, scholarships allow for enough prices to induce social efficiency. Epple and Romano [1998] derive similar results and simulate some voucher experiments. The results of these studies apply here directly.

26. Private schools with $A$'s should have larger classes and should be cheaper than those with $B$'s. This is surely true, at least at the extremes. Private schools for special needs children have small classes and are very expensive (although they are sometimes subsidized by the state).

outside $B$. Therefore, matching student types both is efficient and is the outcome of a competitive bidding process.

Public $K$–12 schools use neither type-specific prices nor admissions criteria to sort students. The implicit price of $K$–12 public schools does vary through housing prices and local taxes. Furthermore, these indirect price variations appear to sort students, albeit imperfectly.[27] The within-school variation in educational attainment, even in public schools, is small relative to the total variation. Some of this may be a result of the schooling itself, and some is a result of the characteristics of the underlying student bodies.

### B. The Case for Integration

Some educators believe that it is important to have integrated classes, where tracking by ability does not occur early, if at all. There are two arguments for integration of classes, in light of Proposition 3: efficiency and equity.

The efficiency case rests on the ability to transform low $p$ students into high $p$ students as a result of integration. To make the case, it is necessary that $B$'s can be *transformed* into $A$'s by being around them. If this effect is strong enough, then integrated classes are efficient.[28] For example, if $B$'s were immediately transformed into $A$'s when integrated with them, and if this imposed no cost on $A$'s, then efficiency would be enhanced by mixing $B$'s with $A$'s.

As a practical matter, transformation of $B$'s into $A$'s is most likely to occur when the ratio of $A$'s to $B$'s is large. If a school of 100 had 99 $B$'s and 1 $A$, it is unlikely that the one $A$ student would change the behavior of all of the other $B$ students.[29] Clear evidence of the effect of peer group on transforming behavior is presented by Katz, Kling, and Liebman [1999]. They find that children who move from high poverty areas to higher income areas experienced reduced incidence of behavior problems, including those at school. The effect was significant for boys, but not

27. See Hoxby [1998].

28. Interestingly, there is an increasing trend toward using other factors, one of which is community service, as a criterion for admission to college. Perhaps colleges believe that those $A$'s who engage in community service are also likely to encourage $B$'s to behave like $A$'s.

29. Lazear [1999] presents a model of cultural acquisition. There, it is argued that incentives to become assimilated into the majority culture depend on the size of the relevant groups. The smaller is the minority relative to the majority, the greater is the incentive of a minority member to acquire the culture of the majority.

for girls. Equity issues may also be at play. Even if it is more efficient to segregate schools, $B$'s bear the costs and are poorer than $A$'s, even without segregation. Segregating classes exacerbates income inequality because $A$'s benefit from segregation and $B$'s may lose by it.[30] This follows directly from (3) and (4). Note that a $B$ receives $p_B^n$ in an all $B$ class, which can be rewritten as $p_B^{\alpha n} p_B^{(1-\alpha)n}$. Now, $p_B^{\alpha n} p_B^{(1-\alpha)n} < p_A^{\alpha n} p_B^{(1-\alpha)n}$ because $p_A > p_B$. But $p_A^{\alpha n} p_B^{(1-\alpha)n}$ is what $B$'s receive in a mixed class, so $B$'s receive more in a mixed class than in an all $B$ class. Similar logic reveals that $A$'s receive more in mixed classes than in an all $A$ classes. Thus, moving to integrated classes reduces educational inequality.

In addition to the results by Katz, Kling, and Liebman [2001], Betts and Shkolnik [1999] find that the percent of time spent on instruction rises and that on discipline falls as the class becomes more female. Following the logic above, boys would want to be in all-girl schools, but girls would not want them there.[31]

## IV. Endogenous Discipline, Teacher Quality, and Other Issues

### A. Endogenous Discipline

Disruption has been assumed to be given, but it is clear that discipline can affect the level of disruption, for better or worse. Catholic schools are known for strict discipline, and some attribute the success of their educational programs to discipline. Discipline, however, is not without a cost. In addition to stifling potential creativity, the act of disciplining students is time-consuming and unpleasant.

The choice of discipline level can be modeled, and $p$, the probability of behaving, can be made endogenous. Let

$$p = p(d,t),$$

where $d$ is the amount of discipline and $t$ is the student's type. It is reasonable to assume that more discipline results in less dis-

30. This is a somewhat controversial point. It is possible that even $B$'s do better by being in segregated classes. The questions that $A$ students ask may be disruption as far as $B$'s are concerned, if the questions are so far above the $B$ level as to render them time-wasters.

31. Hoxby [2000b] finds that female classes perform better in the lower grades and that gender composition alters classroom conduct. There is a recent push to create girls-only classes in the public schools. It is important to define "disruption" in this context. Males who interrupt their teachers more might be more engaged in the class, raising the value of the educational experience. The optimal amount of student participation is not zero.

ruption so $p_1 > 0$. Also, $t$, the student's type is defined such that $p_2 > 0$; that is, higher $t$ students are better-behaved students. Standard concavity assumptions are that $p_{11} < 0$ and $p_{22} < 0$. Finally, assume that better-behaved students are more heavily affected by a given amount of discipline or that $p_{12} > 0$. Discipline occurs only when a disruption is initiated, so total discipline in a class of size $n$ is

$$nd[1 - p(d,t)^n].$$

Now, let there be a cost of imposing discipline on a student given by $c = hc(d)$, with $c', c'' > 0$ where $h$ is a shifter reflecting different costs of discipline. Given that $p$ can be affected by discipline, one can rewrite (1a) as

$$(1') \qquad V(p(d,t))^n - \frac{W}{n} - \frac{(1 - p(d,t)^n)hc(d)}{n}.$$

The first-order conditions are then

$$(2'\text{a}) \quad \frac{\partial}{\partial n} = Vp^n \ln(p) + \frac{W}{n^2} + \frac{p^n \ln(p)hc(d)}{n} + \frac{(1 - p^n)hc(d)}{n^2} = 0$$

and

$$(2'\text{b}) \quad \frac{\partial}{\partial d} = Vnp^{n-1}p_1 - \frac{(1 - p^n)hc'(d)}{n} + p_1p^{n-1}hc(d) = 0.$$

Using (2′b),

$$\frac{\partial d}{\partial t}\Big|_{(2'b)} = -\frac{[\partial(\partial/\partial d)]/\partial t}{\partial^2/\partial d^2}$$

which equals

$$\frac{\partial d}{\partial t}\Big|_{(2'b)} = -\frac{[(n-1)p^{n-2}p_1p_2 + p^{n-1}p_{12}][nV + hc(d)] + p^{n-1}p_2hc(d)}{\partial^2/\partial d^2}.$$

The numerator is positive, and the denominator negative, so the expression is positive. Better behaved students encounter more discipline per infraction because the positive effects of discipline on their behavior are greater.[32]

32. This does not imply that there is more discipline in classes where students are inherently better behaved. Because $p$ increase in $t$, total discipline may be higher or lower in classes with better (higher $t$) students because students encounter discipline $(1 - p)$ of the time.

Catholic schools may obtain better outcomes for two reasons, both related to discipline.[33] It has already been shown that the pricing structure automatically induces better-behaved, higher $t$, students to attend Catholic schools. Since $\partial d/\partial t$ is positive, Catholic schools should use more discipline per infraction. Second, the political constraints that public schools face relative to private schools may make it more costly to discipline students in a public school. Formally, this means that $h$ is higher in the public schools. Under usual conditions, the effect of raising $h$, the cost of discipline, is that the amount of discipline used will fall.[34]

Discipline, learning, and class size have been linked empirically. Betts and Shkolnik [1999] find a significant positive effect of class size on the amount of time spent on discipline and a negative effect of class size on the amount of time spent in instruction. Currie and Thomas [1996] find that discipline has negative effects on test scores, although the results are imprecise.[35] Grogger [1997] finds that an extreme form of disciplinary problem, namely violence at school, has negative effects on high school graduation and college attendance. The model presented gives precise predictions on the relation of disciplinary problems to class size.

Strict discipline is a substitute for small class size, given the production technology postulated. For any given level of educational output per head, $X$, there is always a trade-off between class size and $p$. The functional relationship between class size and discipline is very simple. In order to increase class size by a factor of $k$ and keep educational output per student constant, it is necessary to improve discipline such that $p$ rises to $p^{1/k}$.[36] For

33. Grogger and Neal [1999] find that Catholic schools raise performance primarily for urban minority students. If urban minority students are in public school environments where discipline is a particular problem, then one would expect effects of shifting to Catholic schools to be largest in such environments.

34. Required is that $(1 - p^n)c'/n > p_1 p^{n-1} c$.

35. Rosen [1987] argues that teaching is a labor-intensive service that does not lend itself to mass production. This can be interpreted in this context as saying that what one student needs to know another does not, which can be interpreted as $p < 1$. When the teacher is addressing the specific needs of one student, the rest of the class is not benefiting, or at least not benefiting by as much as they would if their personal needs were addressed.

36. To see this, let $p_1$ be initial $p$, $p_2$ be the new $p$. To keep educational output constant while changing class size by a factor of $k$, it is necessary that

$$p_1^n = p_2^{nk}$$

or

$$p_2 = p_1^{1/k}.$$

example, in a class of 25, where $p = .98$, educational activity occurs 60 percent of the time. To double class size and obtain the same level of educational output, it is necessary to raise $p$ from .98 to $\sqrt{.98}$, which equals .99. Thus, each student must behave 1 percent more of the time. Although this may seem like a relatively minor improvement in behavior, the statement can be turned around: the amount of misbehavior must be cut from 2 percent of the time per student to 1 percent of the time per student. This implies that a halving of misbehavior is necessary to effect a constant educational output while doubling class size. Whether this is large or small depends on student responsiveness to disciplinary incentives, and this cannot be stated a priori.

Discipline is one way to produce higher $p$ in the classroom. Another may be to promote a particular classroom etiquette. Although students are generally allowed to ask questions in class in college and graduate courses, questions are generally discouraged in large lecture classes having a few hundred students. Etiquette varies directly with class size. As class size increases to numbers like 500, $p$ needs to approach 1 for there to be any educational output at all. Of course, neither discipline nor etiquette comes without cost. If it were free to produce high levels of $p$, then all classes would consist of an extremely large group of passive and silent students. At some point, the learning component suffers when questions are prohibited.

### B. *Teacher Quality*

Classroom behavior, captured by $p$ or $p^n$, may be as much a function of the teacher as it is of student characteristics. A given student is more attentive and has fewer off-the-mark questions in a good teacher's class than in a poor teacher's class. Just as there is a role for discipline in raising $p$, there is a role for altering teacher quality. Hanushek and Rivkin [1999] argue that teachers are the most important determinants of educational output.

Teacher quality can be raised by paying higher salaries. To the extent that labor supply to the profession is upward-sloping, higher salaries imply a larger pool of applicants, which permits a school to engage in more selective hiring. How much is this worth? The effect of raising teacher quality, even through substantial pay increases can be impressive. To see this, consider the following numerical example.

Consider a school of 100 students, where $p = 0.97$, and teacher's wage, $W, = 5$. Optimal class size is then 16.5 students, learning occurs 61 percent of the time, and the profit (or net social value) in this school is 30.2. Suppose that teacher salary is doubled and that the effect is to raise $p$ from 0.97 to 0.99. Then, optimal class size rises to 25 students, learning occurs 78 percent of the time, and the profit in this school rises to 57.8.

A doubling of teacher salary is quite a dramatic increase. Using 1999 CPS data, this would mean an increase in average teacher salary from \$32,300 to \$64,600, which would put teachers near the eighty-fifth percentile of college grads, and well above the median college graduate who earns \$36,000 per year. Whether this kind of selectivity could bring about an increase in $p$ of at least 2 percent is an empirical question, but the example makes clear the potential power of teacher quality in affecting outcomes.

### C. Movers

Using data from Texas, Hanushek, Kain, and Rivkin [1999] find that children who switch schools perform more poorly than those who do not and that moving imposes costs on other students. Both can be interpreted in the context of the model presented above. Since movers are unaccustomed to their new classroom, their questions are more likely to be disruptive in that they relate to material that the initial class members have already covered. The movers' $p$ is low relative to the nonmovers, which implies that movers' own learning is slower than it would have been if they had not moved. It also implies that because movers lower the average $p$ for the class, learning by others is reduced as well.

### D. An Empirical Strategy

The fraction of the time that a student is not an initiator of disruption, denoted $p$, is not a mere abstraction, but is observable. One could imagine obtaining information on $p$ by surveying teachers or by actually observing a classroom. Given $p$, quality of education should vary with $p^n$. Since $n$ is also observable, $p^n$ can be thought of as a measure of quality that is different from educational expenditures used by others.[37] A year of adjusted schooling could be defined as a year, multiplied by $p^n$.

The model has very specific predictions about the relation of $n$ to $p$. The first-order condition in (2) implies that class size

37. See, for example, Card and Krueger [1992].

should be smaller, the larger is $p$. This is testable using data such as TIMSS, where the classroom experience is videotaped, reviewed, and graded.[38]

Operationally, it is probably easier to observe $p^n$ than $p$. An alternative to direct viewing of classrooms is surveying the teachers on the proportion of their class time spent in actual teaching versus discipline or disruption. The Longitudinal Study of American Youth provides information on time spent on learning and discipline as reported by teachers.[39] These data provide an estimate of $p^n$, which when coupled with information on $n$, provide an estimate of $p$. For the purposes of quality adjustment, $p^n$ by itself is all that is of interest. For normative purposes, for example, determining the optimum class size by grade level, $p$ is useful. It might also be of interest to compare student characteristics with $p$. How does $p$ vary with age, socio-educational background, and parent's income? Understanding variations in $p$ may provide some implications for school reform.

## V. An Alternative Model of Class Size

The disruption model emphasizes teaching technology as a determinant of class size. There is another possible explanation of why classes are not larger than they are. Class size may be limited by the extent of the market. Consider, for example, a small college that has students in both literature and economics. Suppose that the college has 100 students, half of whom are in each field. On the basis of congestion effects, the condition in (2) might imply an optimal class size of 100, which is infeasible because only 50 want to study each field. Heterogeneous study preferences limit class sizes beyond congestion considerations. It is for this reason that schools do not put high school seniors in the same class with first-graders. Absent other constraints, one would conclude that a school of 100 is too small. Because class sizes must be below the efficient number of students, a merger of two small schools into one larger one could achieve both division of labor and class size efficiency. To the extent that a desire for

38. The TIMSS (Third International Mathematics and Science Study), conducted in 1995–1996 is a survey of about one-half million students from 41 countries in three grades to determine math and science achievement.

39. The LSAY is a national study of student interests and aptitudes in math and science. The centerpiece is a massive longitudinal survey of schools, students, parents, and teachers from 1986–1994.

neighborhood schools or other preferences limit the size of a school, heterogeneous learning preferences limit the size of classes.

That having been said, a limited extent of the market cannot be used to explain a number of facts that are consistent with the congestion hypothesis. First, congestion implies that schools may have four identical classes of 30 students at a particular grade level, whereas heterogeneous preferences would imply one class of 120. Second, heterogeneity would argue for smaller classes among older students, not the reverse. The curriculum for preschool students is more homogeneous across students than is the curriculum for high school students who are following different paths. Preschool classes are smaller than high school classes, not because different courses are being offered, but because things get out of control when there are more than ten preschoolers in the same place. Similarly, special education classes are small not because the course material is so varied, but because $p$ is low for special students.

## VI. Conclusion

Classroom teaching is a public good. As such, congestion effects can be important. A student who is disruptive or who takes up teacher time in ways that are not useful to other students affects not only his own learning, but that of others in the class. It is for this reason that class size may have important effects on educational output. Much of the empirical evidence, however, suggests otherwise. Class-size effects are small or nonexistent in most studies.

A theory of educational production, with particular emphasis on classroom dynamics, has been presented. The model, which offers direct implications about the choice of class size as a function of student characteristics, is consistent with a large variety of findings in the education literature. The empirical literature has wide-ranging findings on the relation of educational output to class size. The model implies that better students are optimally placed in larger classes, and further, that educational output is higher in the large classes, despite the reduced teacher-student ratio. The disruption framework provides a specific model of class-size endogeneity that can be tested, verified, or refuted. There is already a great deal of evidence with which the model is consistent, although there remain some countervailing pieces of evidence. The theory provides implications for class size and its effect on total output. The analysis predicts variations in class size by grade level and by other student and teacher characteristics. In equilibrium, class size matters very little. To the extent that class size matters, it is more likely to matter at lower grade levels than at upper grade levels where class size is smaller.

The technology implies that class-size effects are more pronounced in smaller classes and for lower values of $p$, which implies that class-size reductions provide better results for disadvantaged and special needs children.

Discipline is a substitute for class size. The structure provides an exact relation of disruption to class size. Specifically, educational output per student remains constant when class size is increased by a factor of $k$ as long as the proportion of the time that students behave rises from $p$ to $p^{1/k}$.

Under most circumstances, segregating students by academic ability maximizes total educational output. Self-selection induces the more attentive students to attend private schools. This mechanism implies that students who opt for Catholic schools (at a positive price) are inherently more attentive than those in public schools. It also means that discipline should be more intense in Catholic schools which would lead them to outperform public schools.

Teachers may prefer smaller classes either because wages do not reflect working conditions fully or because teachers as a group can raise the demand for their services by lowering class size. However, in a competitive labor market, where teachers' wages depend on job attributes, there is no tension between teacher preferences and those of students or their parents.

## Appendix

*Proof of Proposition 1*

Using the implicit function theorem on (2), note that

$$\frac{\partial^2 \text{profit}}{\partial m^2} = VZ^2 p^{Z/m} \ln(p) \frac{2m + Z \ln(p)}{m^4},$$

which is negative for the solution to be an interior one. This implies that

$$2m + Z \ln(p) > 0.$$

Next, because $\partial/\partial W = -1$,

$$\frac{\partial m}{\partial W}\Big|_{f.o.c.} = \frac{1}{\partial^2\text{profit}/\partial m^2} < 0.$$

Also,

$$\frac{\partial m}{\partial p}\Big|_{f.o.c.} = \frac{-VZ^2 p^{(Z-m)/m}(m + Z\ln(p))/m^3}{\partial^2\text{profit}/\partial m^2} < 0$$

for interior solutions, which are guaranteed for $p$ near 1. Further,

$$\frac{\partial m}{\partial V}\Big|_{f.o.c.} = \frac{(Z^2/m^2)p^{Z/m}\ln(p)}{\partial^2\text{profit}/\partial m^2} > 0.$$

Also,

$$\frac{\partial m}{\partial Z}\Big|_{f.o.c.} = \frac{VZp^{z/m}\ln(p)(2m + Z\ln(p))/m^3}{\partial^2\text{profit}/\partial m^2} = \frac{m}{Z} > 0$$

and

$$\frac{\partial m}{\partial Z}\frac{Z}{m} = 1,$$

which is as expected because no class-size–school-size interactions are built into the model.

Now, since $m = Z/n$, $dn/dm = Z/m^2$, so $\partial n/\partial W > 0$, $\partial n/\partial Z < 0$, $\partial n/\partial V < 0$, and $\partial n/\partial p > 0$.‖

*Proof of Proposition 2*

At the optimum $m$, educational output-per-student decreases in $p$.[40] First, without loss of generality, normalize $Z$ and $V$ to 1. Then, denote profit as

$$\pi = x(p,m(p)) - Wm(p),$$

where

$$\text{(A1)} \qquad x(p,m(p)) = p^{1/m(p)}.$$

The first-order condition says that

$$x_2(p,m(p)) = W.$$

Totally differentiating with respect to $p$ gives

$$\text{(A2)} \qquad x_{12} + x_{22}m'(p) = 0.$$

Now, differentiating $x()$ with respect to $p$ yields

40. Simon Board provided this proof.

(A3) $$\frac{\partial x}{\partial p} = x_1 + x_2 m'(p).$$

Substituting (A2) into (A3) yields

(A4) $$\frac{\partial x}{\partial p} = x_1 - x_2 \frac{x_{12}}{x_{22}}.$$

Using the definitions in (A1) and applying them to (A4) yields

$$\frac{\partial x}{\partial p} = \frac{1}{m} p^{1/m-1} + \frac{1}{m^2} p^{1/m} \ln(p) \frac{(-p^{1/m-1}/m^3)[\ln(p) + m]}{(p^{1/m}\ln(p)/m^4)(2m + \ln(p))}$$
$$= \frac{p^{1/m-1}}{2m + \ln(p)} > 0$$

because $2m + \ln(p) > 0$ by the second-order condition and $p$ is positive. Thus, educational output rises in $p$.‖

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